Radial Consolidation Analysis by the Steepest Slopes Observed in the δ_t - $\log t$ and δ_t - $\sqrt{t}$ Curves

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Abstract— **The steepest slopes method for radial consolidation is proposed herein for estimating the coefficient of radial consolidation c_r, which is directly related to the ratio of the steepest slopes observed in the compression-log time (δ_t - $\log t$) and in the compression-root time (δ_t - $\sqrt{t}$) curves. The proposed method involves only the identification of two straight lines with steepest slopes and compute c_r without the need to identify the initial or secondary compression. Accordingly, the proposed method does not require a complete record of compression - time data; few data points at middle of primary consolidation range are adequate to estimate c_r particularly in the case of field conditions. Because the proposed method computes c_r in the middle of primary consolidation range, the computed c_r is least affected by the initial and secondary compressions. Experimental results of radial consolidation tests are used to validate the proposed method by comparison with existing methods and by matching experimental compression - time data with the theory**.

***Index Terms*— Barron's theory, steepest slopes, radial drainage, coefficient of consolidation, EOP settlement, inflection point.**

I. INTRODUCTION

Vertical sand and prefabricated drains along with surcharge loads are commonly used for improvement of soft clay deposits so as to increase their bearing capacity and to reduce settlement of these deposits. Because of its simplicity, the Barron theory [1] is widely used for the design of vertical drains. Barron [1] provided solutions for two types of boundary conditions; that is, the free strain conditions resulting from a uniformly distributed loads and equal strain conditions resulting from imposing uniform settlement. Vertical drains accelerate the slow consolidation process of soft clay deposits by providing shorter horizontal drainage path and by utilizing the fact that the horizontal permeability is generally higher than the vertical permeability.

The application of the Barron consolidation theory that was developed for evaluating settlement and settlement rate of clay deposits using vertical drains requires a rational assessment of the coefficient of radial consolidation c_r and end-of-primary (EOP) settlement δ_p. The Barron theoretical relationship between the degree of radial consolidation U_r and time factor T_r was used to estimate c_r and δ_p [2-11]. Generally, different c_v and δ_p values may be obtained by these existing methods; this difference may be attributed chiefly to the influence of initial and secondary compressions. All these existing methods compare specific characteristic features of the Barron theoretical U_r - T_r relationship with the experimental settlement - time (δ_t - t) data interpreted in different ways to obtain the EOP settlement and coefficient of radial consolidation.

The two standard methods, namely, the δ_t - $\log t$ method [12] and δ_t - $\sqrt{t}$ method [13], are commonly used for the determination of the coefficient of vertical consolidation c_v. The δ_t - $\log t$ method [12] requires the determination of both the initial and final compressions that correspond to 0 % and 100 % consolidation, respectively, to compute c_v at 50 % consolidation. The δ_t - $\sqrt{t}$ method [13] requires the determination of the initial compression that correspond to 0 % consolidation and the initial linear portion of the δ_t - $\sqrt{t}$ curve to compute c_v at 90% consolidation. On the other hand, the δ_t - $\log t$ method [9] and δ_t - $\sqrt{t}$ method [4] were developed for the determination of the coefficient of radial consolidation c_r such that these two methods can be used in a similar way to the two standard methods of vertical consolidation described above.

In this study, the steepest slopes method for radial consolidation (SSM-RC) under equal vertical strain conditions is proposed for estimating the coefficient of radial consolidation c_r using characteristic features of both standard δ_t - $\log t$ and δ_t - $\sqrt{t}$ curves in the primary consolidation range without the need to estimate the initial or secondary compression and without the explicit use of any specific U_r value. The proposed method involves only identification of two straight lines with the steepest slopes of the δ_t - $\log t$ and δ_t - $\sqrt{t}$ curves; the ratio of these steepest slopes is directly used to compute c_r. The proposed steepest slopes method is directly linked to other existing methods; for example, the proposed methods can indirectly be used to identify the inflection point and the corresponding time in both standard δ_t - $\log t$ and δ_t - $\sqrt{t}$ curves. Experimental results on vertical and radial consolidation tests on several clayey soils are used to validate and compare the proposed method with existing methods.

II. STEEPEST SLOPES METHOD FOR RADIAL CONSOLIDATION (SSM-RC)

The Barron consolidation theory [1] expressed in terms of U_r and T_r is often used to determine c_r required for evaluating settlement and settlement rate of soil deposits pre-consolidated under radial drainage using vertical drains. The Barron U_r - T_r relationship for equal vertical strain conditions can be given as follows

$$U_r = 1 - \exp\left(-\frac{8T_r}{F(n)}\right) \tag{1}$$

where

$$F(n) = \frac{n^2 \ln(n)}{(n^2-1)} - \frac{3n^2-1}{4n^2} \tag{2}$$

$$n = \frac{D_e}{d_w} \tag{3}$$

$$T_r = \frac{c_r t}{D_e^2} \tag{4}$$

$$U_r = \frac{\delta_t}{\delta_p} \tag{5}$$

The n value represents the drain spacing influence ratio defined as the ratio of the equivalent diameter of influence (D_e) to the diameter of the drain well (d_w).

Characteristic features of the Barron theoretical U_r - T_r relationship and experimental δ_t - t data plotted in different forms were used by the existing methods to compute c_r. In the following analysis, a steepest slopes method for radial consolidation (SSM-RC) is developed based on the Barron theory to estimate c_r; the characteristic feature of this method is to use the steepest slopes at the two inflection points observed in the standard δ_t - $\sqrt{t}$ and δ_t - $\log t$ curves.

A. *Steepest slope at inflection point observed in the U_r - $\sqrt{T_r}$ curve for radial drainage*

Fig. 1 shows that an inflection point is observed in all theoretical U_r - $\sqrt{T_r}$ curves at the same degree of radial consolidation of 39.3% independent of n (i.e., drainage pattern) [10]. The steepest tangential slope at the inflection point can be obtained by using the first and second derivatives of Eq. 1 with respect to $\sqrt{T_r}$ (where the second derivative of U_r with respect to $\sqrt{T_r}$ is zero) and can be expressed as follows

$$\left(\frac{dU_r}{d\sqrt{T_r}}\right)_{IP} = \frac{2.428}{\sqrt{F(n)}} \tag{6}$$

Substituting Eq. 4 and 5 into 6, the steepest slope $m_{\sqrt{IP}}$ at the inflection point observed in the experimental δ_t - $\sqrt{t}$ curve can be given in terms of c_r and δ_p as follows

$$m_{\sqrt{IP}} = \left(\frac{d\delta_t}{d\sqrt{t}}\right)_{IP} = \delta_p \frac{2.428}{\sqrt{F(n)}} \sqrt{\frac{c_r}{D_e^2}} \tag{7}$$

Therefore, the coefficient of radial consolidation can be given in terms of the steepest slope at the inflection point observed in δ_t - $\sqrt{t}$ curve and EOP δ_p as follows

$$\frac{c_r}{D_e^2} = \frac{0.170F(n)}{\delta_p^2} m_{\sqrt{IP}}^2 \tag{8}$$

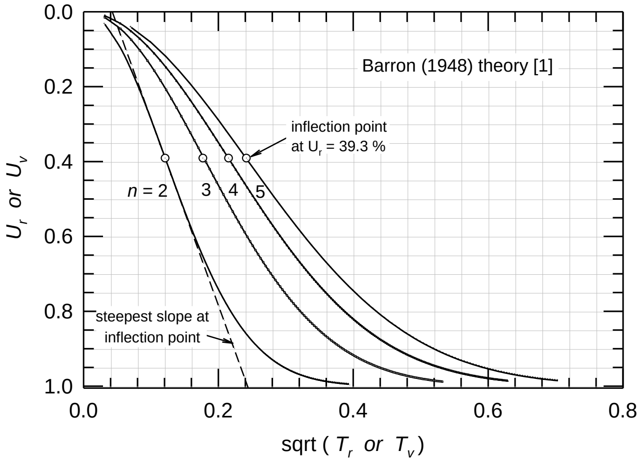

Fig. 1. The Barron theoretical U - $\sqrt{T}$ curves.

Equation 8 implies that the coefficient of radial consolidation can be estimated as long as the EOP settlement δ_p and the steepest slope $m_{\sqrt{IP}}$ at the inflection point observed in the δ_t - $\sqrt{t}$ curve can be obtained. The EOP settlement δ_p is estimated in the following section based on the steepest slope at the inflection point observed in U - $\log T_r$ curve.

B. *EOP δ_p by the steepest slope at inflection point observed in U_r - $\log T_r$ curve*

Fig. 2 shows that an inflection point is observed in the theoretical U_r - $\log T_r$ curves at the same degree of radial consolidation of 63.2% independent of n [5]. The steepest tangential slope at the inflection point observed in the U_r - $\log T_r$ curve can be given using Eq. 1 (where the second derivative of U_r with respect to $\ln T_r$ is zero) as follows

$$\left(\frac{dU_r}{d\ln T_r}\right)_{IP} = 0.368 \tag{9}$$

Substituting Eq. 4 and 5 into 9, the following expression is obtained for the steepest slope $m_{\log IP}$ at the inflection point observed in the δ_t - $\log t$ curve for radial consolidation

$$m_{\log IP} = \left(\frac{d\delta_t}{d\log t}\right)_{IP} = 0.848\delta_p \tag{10}$$

Hence, the EOP settlement δ_p can be expressed as follows

$$\delta_p = 1.18\ m_{\log IP} \tag{11}$$

The EOP settlement δ_p (Eq. 11) is independent of the n value (i.e., drainage pattern) and can be determined by using only the steepest slope $m_{\log IP}$ without the need to use the secondary compression range.

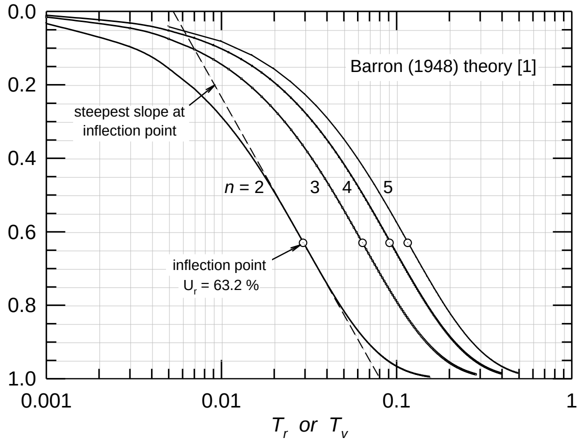

Fig. 2. The Barron theoretical U - $\log T$ curves.

C. The coefficient of radial consolidation

The coefficient of radial consolidation can be expressed, based on Eq. 8 and 11, as a function of the ratio of the steepest slopes at the two inflection points observed in both standard δ_t - $\sqrt{t}$ and δ_t - $\log t$ curves as follows

$$\frac{c_r}{D_e^2} = 0.12F(n)\left(\frac{m_{\sqrt{IP}}}{m_{\log IP}}\right)^2 \qquad (12)$$

Equation 12 implies that the coefficient of radial consolidation can be determined by only the identification of two straight lines with steepest slopes in the δ_t - $\sqrt{t}$ and δ_t - $\log t$ plots at the two inflection points without the direct identification of any of these two inflection points as shown in Fig. 3 and 4. Furthermore, c_r values can also be estimated by the steepest slopes method for radial consolidation (SSM-RC) without the need to identify the initial or secondary compression and without the explicit use of any specific U_r value.

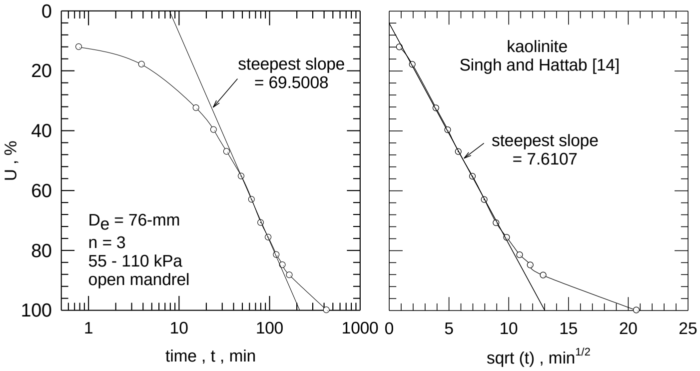

Fig. 3. (a) settlement - log time curve; (b) settlement - root time curve for kaolinite under radial consolidation; data from Singh and Hattab [14].

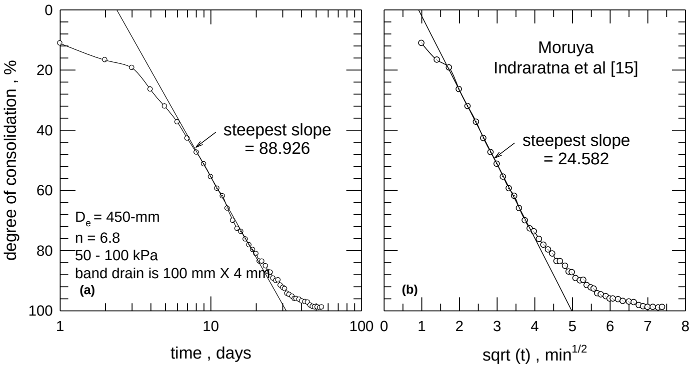

Fig. 4. (a) settlement - log time curve; (b) settlement - root time curve for relatively large sample of Moruyo clay for radial consolidation; data from Indraratna et al. [15].

D. Relationship between $t_{\log IP}$ at the inflection point and steepest slopes

The inflection point method for radial consolidation [5] requires the identification of the inflection point observed at 63.2 % consolidation in the δ_t - $\log t$ curve and the time $t_{\log IP}$ that corresponds to this inflection point; the c_r can be computed by Eq. 2 where $T_{\log IP} = F(n)/8$ is time factor at the inflection point for radial consolidation as follows

$$c_r = \frac{\mathrm{F(n)D}_e^2}{8t_{\log IP}} \qquad (13)$$

On the other hand, the root inflection point method for radial consolidation [10] also requires direct identification of the inflection point observed in the δ_t - $\sqrt{t}$ curve at 39.3 % consolidation and the time $t_{\sqrt{IP}}$ that corresponds to this inflection point; c_r can be computed by Eq. 2 where $T_{\sqrt{IP}} = F(n)/16$ is the time factor at the inflection point observed in the δ_t - $\sqrt{t}$ curve for radial consolidation as follows

$$c_r = \frac{\mathrm{F(n)D}_e^2}{16t_{\sqrt{IP}}} \qquad (14)$$

Therefore, the time $t_{\log IP}$ and $t_{\sqrt{IP}}$ at the inflection points can be expressed by comparing Eq. 4 and 8 as a function of the ratio of the steepest slopes as follows

$$t_{\log IP} = 2t_{\sqrt{IP}} = 1.04\left(\frac{m_{\log IP}}{m_{\sqrt{IP}}}\right)^2 \qquad (15)$$

Fig. 5 shows that this expression (Eq. 15) can reliably be used to estimate the inflection points and the corresponding time in either δ_t - $\log t$ curve or δ_t - $\sqrt{t}$ curve.

The inflection point of the δ_t - $\log t$ curve estimated by Eq. 15 for black cotton clay (Fig. 6a) is about 61.5 % consolidation and for Singapore marine clay (Fig. 7a) is about 65.0 % consolidation; these estimated values are quite comparable to the theoretical value of 63.2 %. Furthermore, the inflection point in the $\delta_t - \sqrt{t}$ curve estimated by Eq. 15 for black cotton clay (Fig. 6b) is about 39.7 % consolidation and for Singapore marine clay (Fig. 7b) is about 41.4 % consolidation; these values are also quite comparable to the theoretical value of 39.3 %. Hence, the use of the ratio of steepest slopes in the proposed method is an alternative to the direct identification of the inflection point and the corresponding time $t_{\log IP}$ or $t_{\sqrt{IP}}$.

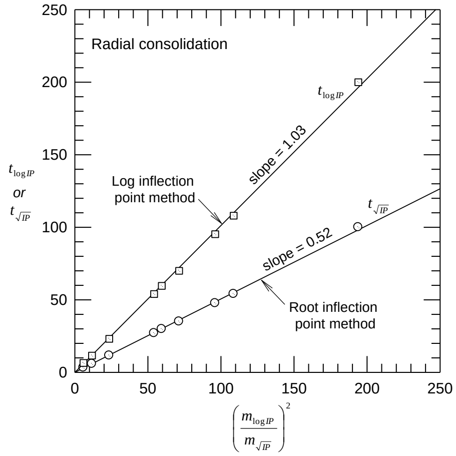

Fig. 5. Comparison of the estimated time at the two inflection points observed in the $\delta_t - \log t$ and $\delta_t - \sqrt{t}$ curve and the ratio of steepest slopes.

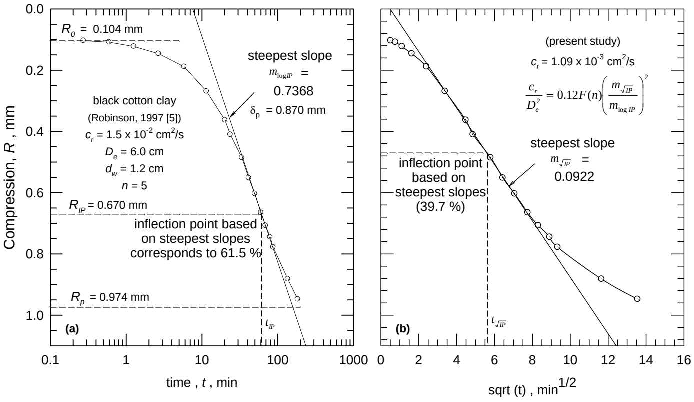

Fig. 6. (a) settlement - log time curve; (b) settlement - root time curve for black cotton clay for radial consolidation without secondary compression; data from Robinson [5].

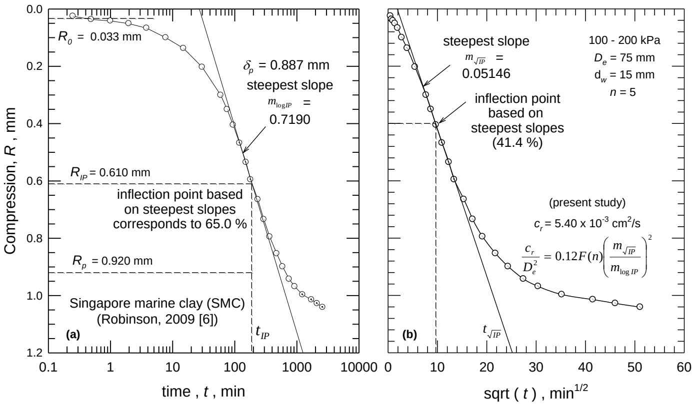

Fig. 7. (a) settlement - log time curve; (b) settlement - root time curve for SMC for radial consolidation with secondary compression; data from Robinson [6].

III. VALIDITY OF THE STEEPEST SLOPES METHOD FOR RADIAL CONSOLIDATION (SSM-RC)

Experimental results of consolidation tests on nine specimens of five clayey soils under radial drainage [5-6, 14-15] are used to validate and compare the steepest slopes method for radial consolidation (SSM-RC) with existing methods as depicted in Fig. 8, which shows that the c_r values of the SSM-RC are quite comparable to those of the one point method [4], log-log method [6], and settlement rate-settlement method for radial consolidation [11] but lower than those of steepest tangent method [7].

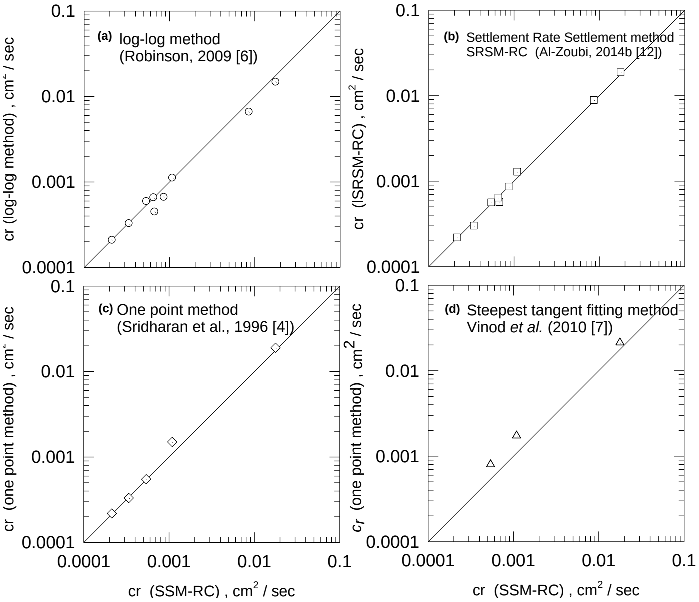

Fig. 8. Comparison of the proposed method (SSM-RC) with four existing methods.

Furthermore, the theoretical settlement - time curve may be expressed in terms of the ratio of the steepest slopes using Eq. 1 to 5 and 15 as follows

$$\delta_t = \left[1 - \exp\left(-0.96\left(\frac{m_{\sqrt{IP}}}{m_{\log IP}}\right)^2 t\right)\right]\delta_p \qquad (16)$$

Equation 16 can also be expressed in terms of $t_{\log IP}$ by the following form

$$\delta_t = \left[1 - \exp\left(-\frac{t}{t_{\log IP}}\right)\right]\delta_p \qquad (17)$$

The theoretical relationships obtained by the proposed steepest slopes method for radial consolidation (Eq. 16) for two samples of kaolinite [14] are plotted in Fig. 9 along with the experimental results of consolidation tests under radial drainage. The theoretical curves match the experimental results well throughout the primary consolidation range.

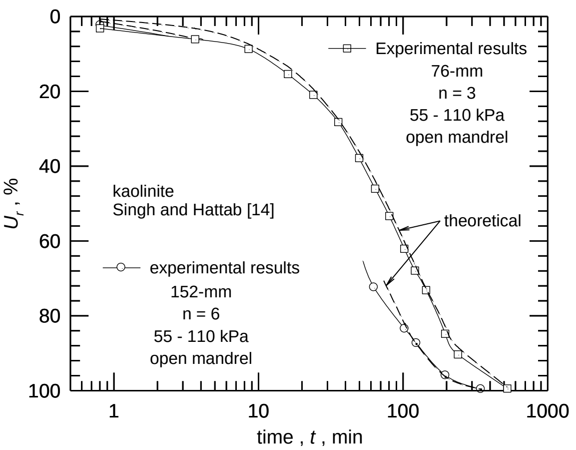

Fig. 9. Comparison of experimental results and theoretical curve obtained by the proposed steepest slopes method using a kaolinite sample (data from Singh and Hattab [14]).

IV. CONCLUSION

The steepest slopes method is presented herein to evaluate coefficient of radial consolidation c_r using characteristic features of both the compression-log time (δ_t - $\log t$) and compression-root time (δ_t - $\sqrt{t}$) curves. The steepest slopes method for radial consolidation explicitly relates c_r to the two steepest slopes observed in the δ_t - $\sqrt{t}$ and δ_t - $\log t$ curves, which are commonly used by the standard existing methods. The proposed method can be considered as an alternative for the inflection point methods [4, 10] but the proposed method does not require direct and careful identification of the inflection points. The proposed method can, however, be used indirectly to identify of the inflection points and the corresponding elapsed times by using the ratio of the steepest slopes. The proposed method does not need to identify the initial or secondary compression; therefore, it does not require a complete record of compression - time data; few data points at middle of primary consolidation range are adequate to estimate c_r by the proposed method particularly in the case of field conditions. Furthermore, the estimated c_r values are least affected by the initial and secondary compressions because the proposed method computes c_r in the middle range of the primary consolidation.

Experimental results of consolidation tests on clayey soils show that the proposed method predicts quite comparable c_r values to those obtained by existing methods [4, 6, 11] but lower than those of the steepest tangent method [7]. However, the proposed method involves only the identification of two straight lines with steepest slopes in the δ_t - $\sqrt{t}$ and δ_t - $\log t$ curves at the inflection points without direct identification of these inflection points and without the explicit use of any specific U_r value.

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