Analytical and Numerical Vibration Study of Unidirectional Hyper Composite Plate

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NOMENCLATURE

ω	Natural frequency of plate.
E_1	Longitudinal moduli for a composite plate.
E_2	Transverse moduli for a composite plate.
E_{cm}	Moduli of isotropic composite matrix, combined of resin and powder.
E_{lf}	Moduli of continuous fiber material.
E_m	Moduli of resin material.
G_{12}	Shear moduli for a composite plate.
G_{cm}	Shear moduli of isotropic composite matrix.
G_{lf}	Shear moduli of continuous fiber material.
G_m	Shear of resin material.
ν_{12}	The major Poisson's ratio for a plate.
ν_{cm}	Poisson's ratio of isotropic composite matrix.
ν_{lf}	Poisson's ratio for continuous fiber material.
ν_m	Poisson's ratio for resin material.
M_x	Moment of Hybrid composite material in x direction.
M_y	Moment of Hybrid composite material in y direction.
M_z	Moment of Hybrid composite material in z direction.
$\forall_{lf}$	Volume fraction of continuous fiber, ratio of the volume of continuous fiber to the volume of composite plate.
$\forall_{pf}$	Volume fraction of powder, ratio of the volume of powder to the volume of composite matrix.
$\forall_{cm}$	Volume fraction of resin matrix, ratio of the volume of resin to the volume of composite plate.
$\forall_m$	Volume fraction of matrix, ratio of the volume of composite matrix to the volume of composite plate.
a	Length of the plate
b	Width of the plate
t	Plate thickness
l_f	Average fiber length
AR	Aspect ratio
ρ	Density of composite plate
ρ_{lf}	Density of continuous fiber.
ρ_{pf}	Density of powder.
ρ_m	Density of resin matrix

***Abstract*--** **An analytical and numerical study of simply supported unidirectional hyper composite plate is presented in this paper. The composite plate is composed of powder glass reinforcement in polyster resinas a matrix and long glass fibers as reinforcement.The analytical method used in this paper to find the general equation of motion is the specially orthotropic plate equations. The Navier solution is applied to solve these equations. A finite element method is used as the numerical method utilizing the ANSYS program ver. 14. The study focused on the effect of the powder and fibers volume fraction on the natural frequency of the plate. The results are so close with maximum error percentage of numerical and analytical about 2.4%, 0.38% and 9.8% for aspect ratio 2, 1 and 0.5 respectively.**

***Index Term*--** **Vibration of plate, vibration of composite plate, vibration ofhyper composite plate, ANSYS plate vibration.**

1- INTRODUCTION

The vibration of composite structure is the important topic of study in nowadays. And has received continued attention in the past few decades. Also it has a large number of applications in several engineering situations. There are a lot of books and researches are handled this topic. The early distinguished solution of the rectangular plate was introduced by Navierin 1823 for simply supported plate. Then Levy in 1899 introduced a solution of a plate withtwo edges simply supported and other two edges with different supporting conditions. A first order theory presented by Yang [1] to calculate the vibration in heterogeneous plate. Whitney [2]shows the effect of the transverse shear on bending of composite plate. After that he [3] shows the shear deformation on heterogeneousanisotropic plates. Srinivas [4,5] study the vibration of simply supported homogeneous and orthotropic plates. Sun [6] studies the theories of dynamic response of laminated plates. Noor [7] study the vibration of multilayered composite plates. Nelson [9] gives a refined theory of orthotropic plates. Bert [10] study the effect of shear deformation on the vibration of anti-symmetric angle ply laminated plates. Reddy [11] presented the vibration of anti-symmetric angle ply laminated plates including transverse shear deformation by finite element method. Murthy [12] gives an improved transverse shear deformation theory for laminated anisotropic plates. Reddy [13] gives a simple higher order theory for laminated composite plates. Then he[14] gives the stability and vibration of isotropic and laminated plates according to higher order shear deformation theory. Putcha [15] studies the stability and natural vibration of laminated plates using mixed element. Pandya [16] gives a finite element evaluation of the higher order shear deformation theory of sandwich plates. Also he [17] gives a finite element stress analysis of composite plates. Kant [18] gives a finite element model for unsymmetric fiber reinforced composite laminate. Mallikarjuna [19] gives refined theories with finite element for free vibration and transient dynamics of anisotropic composite and sandwich plates. Noor [20] studies the stress and free vibration of multilayer composite plates. Kant [21] studies the vibration of unsymmetrically laminated plates by higher order theory and finite element formulation. Manjunatha and Kant [22] gives a comparison of 9 and 16 node element based on higher order laminate theories for estimation transverse stress. Then he [23] presented a paper on accurate estimation of transverse stresses in multilayer laminates. Messina [25] studies the vibration of completely free composite plates and cylindrical shell panels by a higher order theory. Meunier [26] studies free vibration analysis of composite sandwichplates. Narita [27] gives combinations for the free-vibration behaviors of anisotropic rectangular plates under general edge conditions. Kant [28] gives solutions for free vibration of laminated composite and sandwich plates based on a higher-order refined. Ming [29] gives vibration analysis of isotropic and orthotropic plates with mixed boundary conditions. Muhannad [30] gives Analysis of Stiffened and Un-Stiffened laminated Plates Subjected to Time Dependent Loading. Shimpi [31] Studies free vibrations of plate using two variable refined plate theories. Abdul-Razzak[32] studies free vibration analysis ofrectangular plates using higherorder finite layer method. JiuHui[33] gives an exact solution for free-vibrationanalysis of rectangular platesusing Bessel functions. Aydogdu[34] gives free vibrations of anti-symmetricangle-ply laminated thinsquare composite plates. Kumar[35] studies vibration analysis of composite laminated plates using higher-order shear deformation theory with zig-zagfunction. Nabil and Kayser [36] study the effect of crack and cutout on vibration characteristics ofalaminated composite plates using nonlinearfinite element analysis. Eruslu and Aydogdu[37] studies free vibration analysis of short fiber reinforced laminated plates with first shear deformation theory. Adnan and Salam [38] studies free vibration analysis of composite laminated plates using Host 12. According to the above literature, this research showsthe effect of the powder, fiber volume fraction and aspect ratio on the vibration behavior of a hyper composite simply supportedplate with a comparison between the analytical and numerical solutions.

2- Analytical solution for vibration plate

2-1-Mechanical properties of hyper composite plate

The mechanical properties; elastic moduli, density, poisson ratio and volume fraction of the hyper composite plate will be shown in this paragraph.

2-1-1-Composite matrix, (Resin and powder reinforcement):

Spherical fillers are reinforcements associated with polymer matrix. They are in the form of microballs, either solid or hollow, with diameters between 10 and 150 μm. They are made of glass.The composite (matrix + filler) is isotropic, with elastic properties E, G, ν given by the following relations [39],

$$K=\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right],$$

$$E\approx\frac{9.K.G}{3.K+G}$$

$$G=\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right],$$

$$\nu=\frac{1}{2}.\left(\frac{3.K-2.G}{3.K+G}\right)(1)$$

$$E_{cm}=E$$

$$G_{cm}=G$$

$$\nu_{cm}=\nu$$

$$\forall_{cm}=\forall_{p_f}+\forall_m(2)$$

2-1-2-Composite plate, (Resin, powder reinforcement, and long reinforcement fiber) unidirectional Ply:

The mechanical characteristics of the fiber/matrix mixture can be obtained based on the characteristics of each of the constituents. With the definitions in the previous paragraph, one can use the following relations to characterize the unidirectional ply [40],

Modulus of elasticity along the direction of the fiber E_1 is,

$$E_1=E_{l_f}.\forall_{l_f}+E_{cm}.\left(1-\forall_{l_f}\right)(3.a)$$

Modulus of elasticity in the transverse direction to the fiber axis E_2 is,

$$E_2=E_{cm}\left[\frac{1}{\left(1-\forall_{l_f}\right)+\frac{E_{cm}}{E_{l_f}}\forall_{l_f}}\right](3.b)$$

Shear modulus G_{12},

$$G_{12}=G_{cm}\left[\frac{1}{\left(1-\forall_{l_f}\right)+\frac{G_{cm}}{G_{l_f}}\forall_{l_f}}\right]\qquad(3.c)$$

Poisson coefficient ν_{12},

$\nu_{12} = \nu_{l_f}.\forall_{l_f} + \nu_{cm}.\forall_{cm}$(3.d)

$\rho = \rho_{l_f}.\forall_{l_f} + \rho_m.\forall_m + \rho_{p_f}.\forall_{p_f}$(3.e)

By substation the eq.(2) into (3.a to 3.c), get,

$$E_1 = \left(\begin{array}{c} E_{l_f}.\forall_{l_f} + \\ \dfrac{\left[\begin{array}{c} 9.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) * \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}\right]}{\left[\begin{array}{c} 3.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) + \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}\right]}.\left(1-\forall_{l_f}\right) \end{array} \right)$$

(4.a)

$$E_2 = \frac{\left[\dfrac{\begin{array}{c} 9.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) * \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}{\begin{array}{c} 3.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) + \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}\right]}{\left(1-\forall_{l_f}\right) + \dfrac{\left[\dfrac{\begin{array}{c} 9.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right). \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}{\begin{array}{c} 3.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) + \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}\right]}{E_{l_f}}\forall_{l_f}}$$

(4.b)

$$G_{12} = \frac{\left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)}{\left(1-\forall_{l_f}\right)+\frac{G_{cm}}{G_{l_f}}\forall_{l_f}}\text{(4.c)}$$

$$\nu_{12} = \nu_{l_f}.\forall_{l_f} + \frac{1}{2}.\left(\dfrac{\begin{array}{c} 3.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) - \\ 2.\left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}{\begin{array}{c} 3.\left(\frac{E_m}{3.(1-2.\nu_m)}\left[1+3.\left(\frac{1-\nu_m}{1+\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right) + \\ \left(\frac{E_m}{2.(1+\nu_m)}.\left[1+\frac{15}{2}.\left(\frac{1-\nu_m}{4-5.\nu_m}\right).\frac{\forall_{p_f}}{\forall_m}\right]\right)\end{array}}\right).\forall_{cm}$$

(4.d)

3- VIBRATION OF HYPER COMPOSITE PLATE:

Special orthotropic plate equation.

We can determine the expresions for the bending and twisting moments with the displacement and strain fields as in the following equations:

$$U_x = -zw_{,x}$$

$$U_y = -zw_{,y}$$

$$U_z = w \qquad (5)$$

$$\varepsilon_{xx} = -zw_{,xx}$$

$$\varepsilon_{yy} = -zw_{,yy}$$

$$\gamma_{xy} = -2zw_{,xy} \qquad (6)$$

The stresses can be written as,

$$\sigma_{xx} = \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}}\varepsilon_{xx} + \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}\varepsilon_{yy}$$

$$\sigma_{yy} = \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}\varepsilon_{xx} + \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}}\varepsilon_{yy}$$

$$\tau_{xy} = G_{xy}\gamma_{xy} \qquad (7)$$

The stresses can be written as the following:
Substituting for strain equations (6) in (7) to get:

$$\sigma_{xx} = -z\left(\frac{E_{xx}}{1-\nu_{xy}\nu_{yx}}w_{,xx} + \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,yy}\right)$$

$$\sigma_{yy} = -z\left(\frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,xx} + \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,yy}\right)$$

$$\tau_{xy} = -2G_{xy}\, z\, w_{,xy}$$

(8)

The bending moments (per unit length) M_x, M_y and M_{xy} are then determinned as:

$$M_x = \int_{-h/2}^{h/2}\sigma_{xx} z\, dz$$

Then,

$$M_x = -\int_{-h/2}^{h/2} z^2\left(\begin{array}{c}\frac{E_{xx}}{1-\nu_{xy}\nu_{yx}}w_{,xx} + \\ \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,yy}\end{array}\right)dz$$

Finally,

$$M_x = -\left(D_{11}w_{,xx} + D_{12}w_{,yy}\right)$$

$$M_y = \int_{-h/2}^{h/2}\tau_{yy} z\, dz$$

Then,

$$M_y = -\int_{-h/2}^{h/2} z^2\left(\begin{array}{c}\frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,xx} + \\ \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}}w_{,yy}\end{array}\right)dz$$

Finally,

$$M_y = -\left(D_{22}w_{,yy} + D_{12}w_{,xx}\right)$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, z\, dz$$

Then,

$$M_{xy} = -\int_{-h/2}^{h/2} 2G_{xy} z^2 w_{,xy}\, dz$$

Finally,

$$M_{xy} = 2D_{66} w_{,xy} \qquad (9)$$

Where,

$$D_{11} = \frac{E_{xx}\, h^3}{12(1 - v_{xy} v_{yx})}$$

$$D_{22} = \frac{E_{yy}\, h^3}{12(1 - v_{xy} v_{yx})}$$

$$D_{12} = \frac{v_{xy} E_{yy}\, h^3}{12(1 - v_{xy} v_{yx})}$$

$$D_{66} = \frac{G_{xy}\, h^3}{12} \qquad (10)$$

The differential equation for plate is:

$$M_{x,xx} - 2M_{xy,xy} + M_{y,yy} = -q \qquad (11)$$

Substituting for the bending and twisting moments from equation (9) into eq. 11. So the above equations will be:

$$-\left(D_{11} w_{,xx} + D_{12} w_{,yy}\right)_{,xx} - 4\left(D_{66} w_{,xy}\right)_{,} - (D_{22} w_{,yy} + D_{12} w_{,xx})_{,yy} = -q$$

$$D_{11} w_{,xxxx} + 2(D_{12} + 2D_{66}) w_{,xxyy} + D_{22} w_{,yyyy} = q \qquad (12)$$

So the equation of vibration will be as:

$$D_{11} w_{,xxxx} + 2(D_{12} + 2D_{66}) w_{,xxyy} + D_{22} w_{,yyyy} + \rho h\, \ddot{w} = q(x, y, t) \qquad (13)$$

For free vibration:

$$D_{11} w_{,xxxx} + 2(D_{12} + 2D_{66}) w_{,xxyy} + D_{22} w_{,yyyy} + \rho h\, \ddot{w} = 0 \qquad (14)$$

The boundary conditions are:

$$M_x = -\left(D_{11} w_{,xx} + D_{12} w_{,yy}\right) = 0$$

On the edge x=0 and x=a
w=0 on the edge x=0 and x=a
and

$$M_y = -\left(D_{22} w_{,yy} + D_{12} w_{,xx}\right) = 0$$

On the edge y=0 and y=b
w=0 on the edge y=0 and y=b

(15)

The solution of equation (14) satisfying the boundary conditions equation (15) can be written as:

$$w = A\, sin\frac{m\pi x}{a}\, sin\frac{n\pi y}{b}\, sinpt \qquad (16)$$

Where m and n are integers. Substitute the above and its derivatives in equation (14) to get:

$$D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66}) m^2 n^2 \frac{\pi^4}{a^2 b^2} + D_{22}\left(\frac{n\pi}{b}\right)^4 = \rho h^2 \omega_{mn}^2 \qquad (17)$$

Therefor the natural frequancy equation will be:

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h a^4}\left[D_{11} m^4 + 2(D_{12} + 2D_{66}) m^2 n^2 k^2 + D_{22} n^4 k^4\right] \qquad (18)$$

4- Numerical study

The numerical study used in theis research is utilizing the finite element method to find the natural frequancy by using ANSYS ver. 14.The three dimensional model were built and the element type (Solid Tet 10 node 187) were used. The average nodes and elements was approximately about 3281 and 1556 respectively. A sample of meshed composite plate is in shown in Fig. (2). For aspect ratio 1.

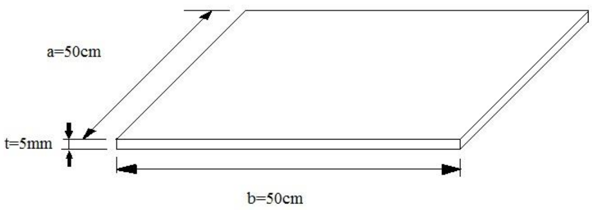

Fig. 1. Composite plate dimensions with AR=1.

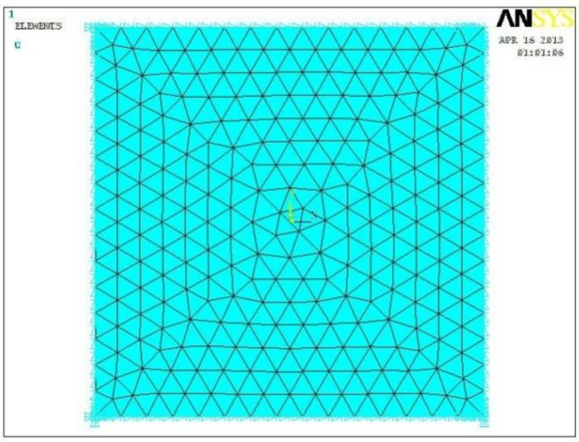

Fig. 2. Meshed composite plate with AR=1.

5- Results and discussions

The natural frequency is calculated for hyper composite plate consisted of powder glass reinforcement in polysterresin and long fibers,with different volume fraction for reinforcement and resin. The properties of these materials are shown in table I.

Table 2 shows the mechanical properties of hyper composite plate for different volume fraction as below.

The natural frequancy is calculated for simple supported hypercomposite plate with dimensions as below, for three aspect ratio,

$a = 50\ cm, t = 5\ mm$, **AR=a/b=0.5 (b=100 cm),**

AR=1 (b=50 cm), AR=2 (b=25 cm)

Figures 3 to 7 for aspect ratio 0.5, the natural frequency is increases as the fiber volume fraction increases. This is because when the fiber volume fraction increases the strength will increase, then the stiffness increases. This will lead to the increasing of the natural freaquency.

Figures 8 to 12 for aspect ratio 1 there is a very good coincidence for numerical and analytical solution. The increase of natural frequency as the fiber volume fraction increses is more than that for aspect ratio of 0.5. this is because that the effect of increasing strength which will lead to increasing the stiffness is more sensible when the plate is square shaped. So the effect of increasing fiber volume fraction have more effect for aspect ratio of 1.

Figures 13 to 17 for aspect ratio 2. When increasing the fiber volume fraction the natural frequancy will decreases, because the effect of the weight will appear in this aspect ratio. The ratio of the stiffness to weight will be small. And the modulus of elasticity is constant, So the increasing in weight will lead to decrease in the natural frequancy.

Figures 18 to 22 shows the natural frequency with respect to long fiber volume fraction for different overall reinforcement voulme fraction(fiber and powder)and aspect ratio.

Figures 23 to 25 shows a contour plot of the changing the natural frequancy with the fiber and powder volume fractions. When the fibers volume fraction increases the natural frequancy increases more than that when powder volume fraction increases. Also one can fined that the type of the powder have noeffect on the natural frquancy but its volume fraction affect the natural frequancy.

6- Conclusion and Recommendation

One can see that the volume fraction of the powder affect the natural frequancy of the plate. But the type of the powder has no effect on the natural frequancy of the plate. Where the volume fraction and the type of the matrix and the longitudinal fiber affect the natural frequancy of the plate.

For future work one can use another type of the fiber and matrix with another many volume fraction for more than one powder material. Also can be use another analytical and numerical method of solutions

TABLE I
DENSITY OF GLASS FIBER AND POLYESTER RESIN MATERIALS.

Materials	ρ (kg/m³)	E (Gpa)	G (Gpa)	ν
Long Fibers	2500	75	30	0.25
Powder Fibers	2750	75	30	0.25
Polyester	1125	3.8	1.4	0.4

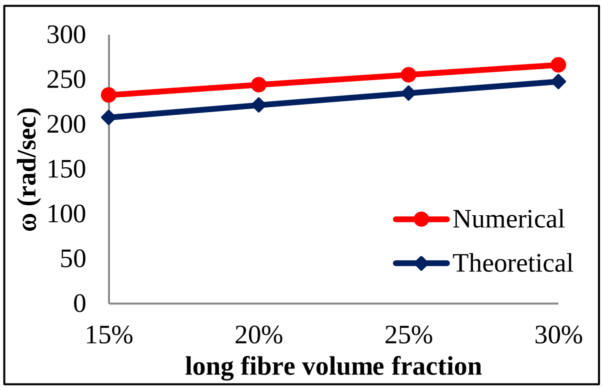

Fig. 3. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=30%, AR=0.5

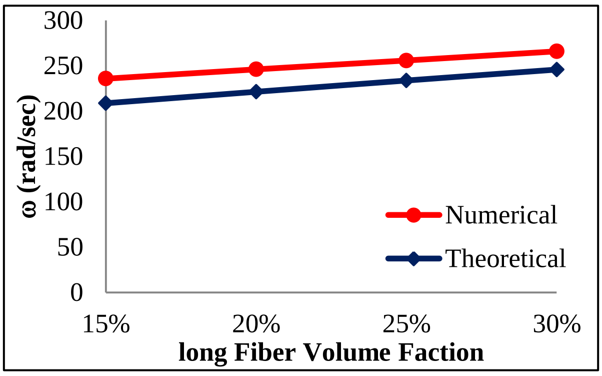

Fig. 4. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder) $\forall_f$=35%, AR=0.5

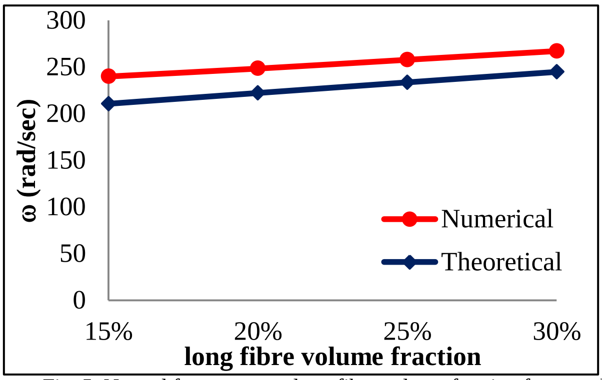

Fig. 5. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=40%, AR=0.5

TABLE II
MECHANICAL PROPERTIES OF HYPER COMPOSITE PLATE

Volume Fraction of Resin and Fiber		Volume Fraction of Long and Powder		E_1 (Gpa)	E_2(Gpa)	G_{12} (Gpa)	ν_{12}	ρ (kg/m^3)
Fiber	**Resin**	**Long**	**Powder**					
30	70	15	15	15.99	6.47	2.34	0.36	1575.00
		20	10	18.99	6.127	2.21	0.36	1562.50
		25	5	22.04	5.74	2.06	0.36	1550.00
		30	0	25.16	5.31	1.90	0.36	1537.50
35	65	15	20	16.64	7.35	2.67	0.36	1656.25
		20	15	19.57	7.00	2.53	0.36	1643.75
		25	10	22.56	6.62	2.39	0.35	1631.25
		30	5	25.61	6.18	2.22	0.35	1618.75
40	60	15	25	17.40	8.37	3.05	0.36	1737.50
		20	20	20.24	8.01	2.91	0.35	1725.00
		25	15	23.15	7.62	2.76	0.35	1712.50
		30	10	26.13	7.19	2.60	0.35	1700.00
45	55	15	30	18.30	9.57	3.49	0.35	1818.75
		20	25	21.04	9.20	3.35	0.35	1806.25
		25	20	23.85	8.80	3.20	0.35	1793.75
		30	15	26.74	8.36	3.03	0.34	1781.25
50	50	15	35	19.37	11.00	4.03	0.35	1900.00
		20	30	21.99	10.62	3.88	0.35	1887.50
		25	25	24.69	10.20	3.73	0.34	1875.00
		30	20	27.47	9.75	3.55	0.34	1862.50

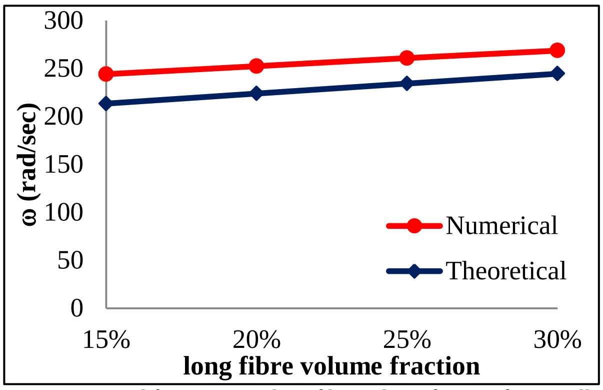

Fig. 6. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=45%, AR=0.5

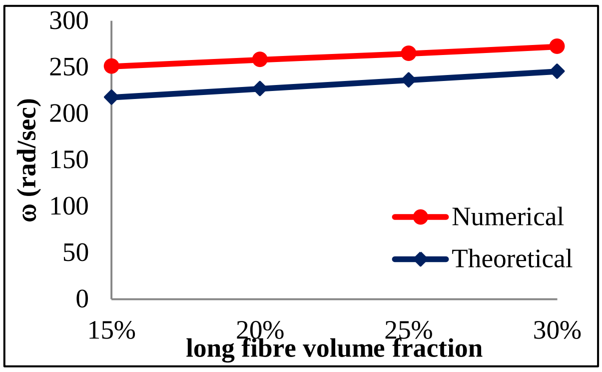

Fig. 7. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=50%, AR=0.5

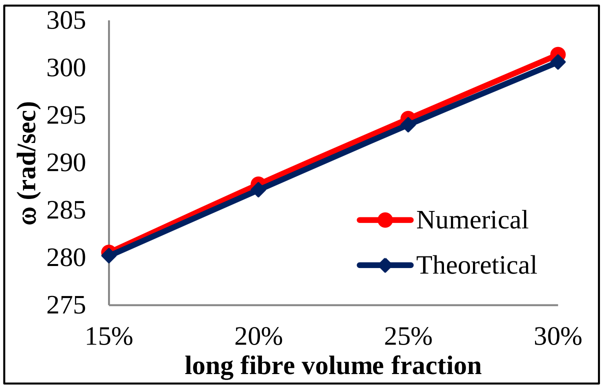

Fig. 8. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=30%, AR=1

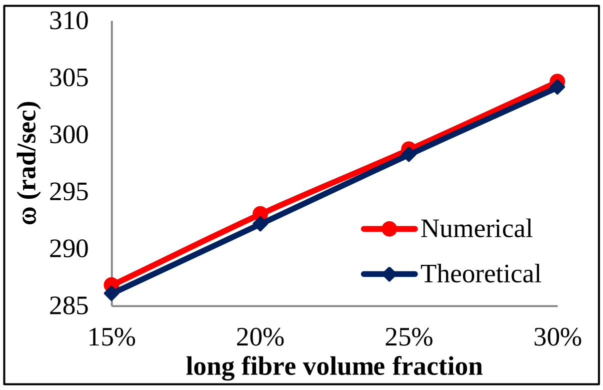

Fig. 9. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=35%, AR=1

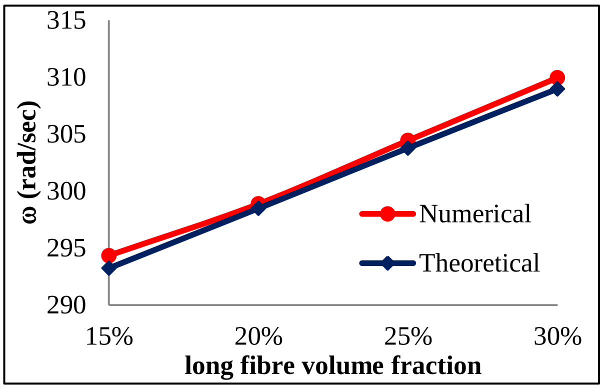

Fig. 10. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=40%, AR=1

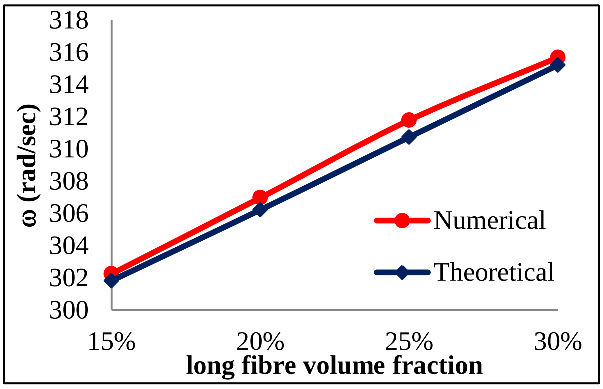

Fig. 11. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=45%, AR=1

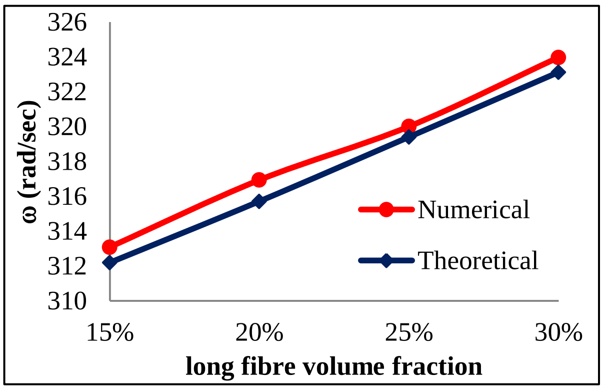

Fig. 12. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=50%, AR=1

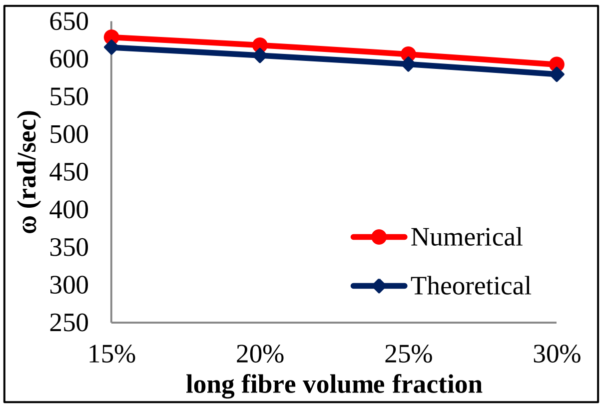

Fig. 13. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=30%, AR=2

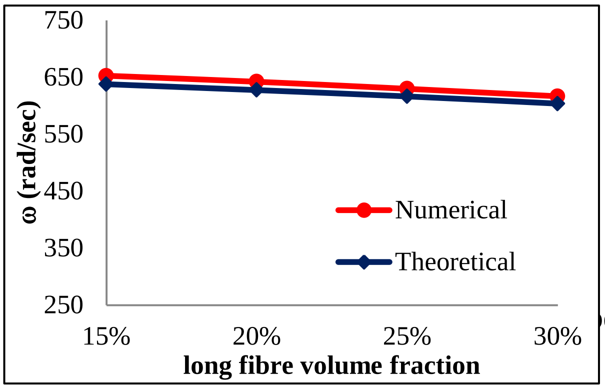

Fig. 14. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=35%, AR=2

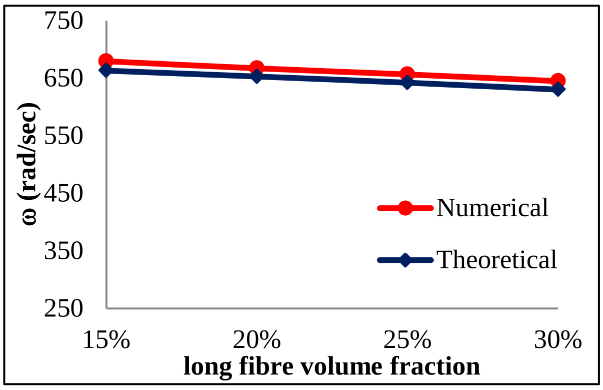

Fig. 15. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=40%, AR=2

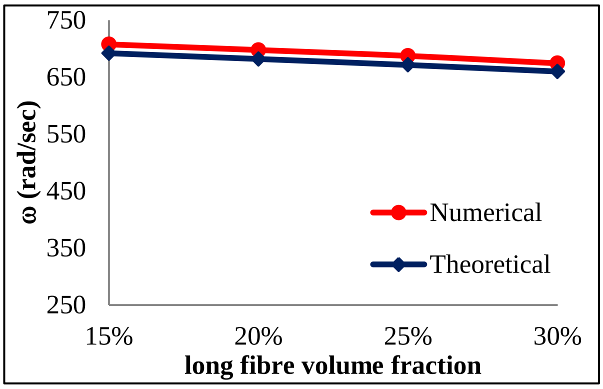

Fig. 16. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=45%, AR=2

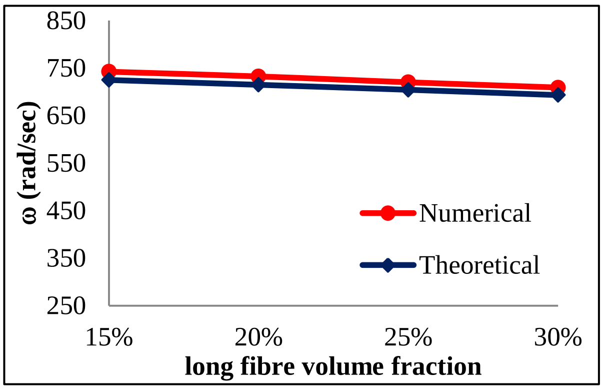

Fig. 17. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=50%, AR=2

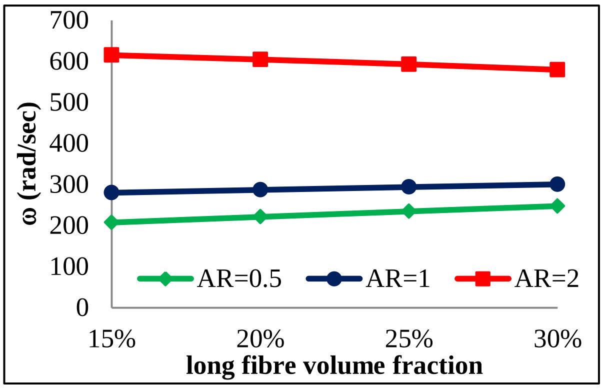

Fig. 18. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=30% for different aspect ratio.

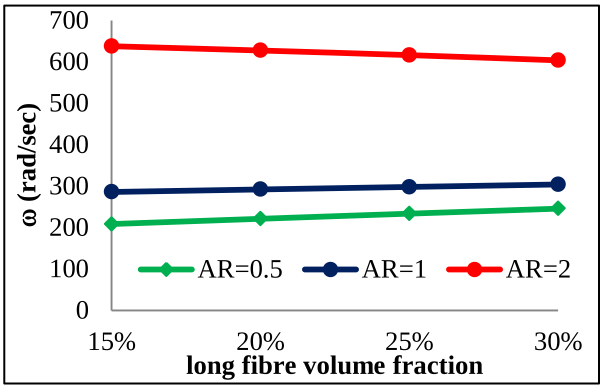

Fig. 19. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=35% for different aspect ratio.

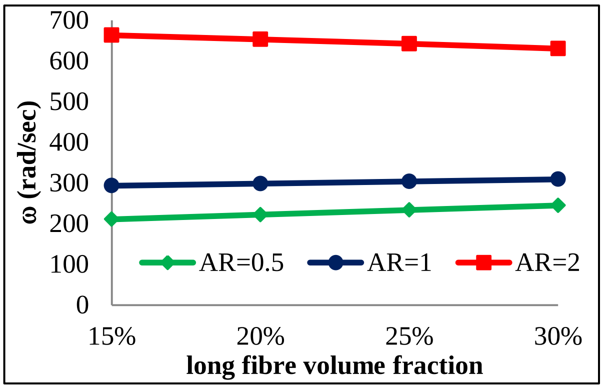

Fig. 20. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=40% for different aspect ratio.

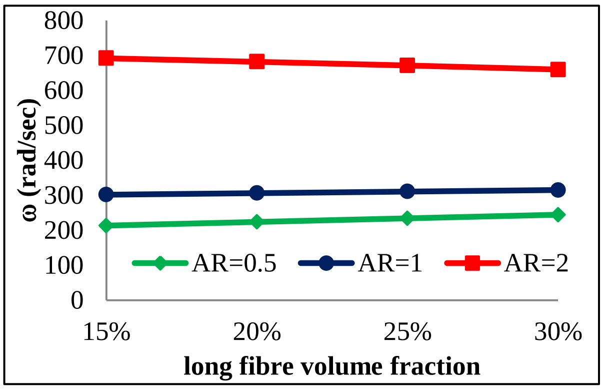

Fig. 21. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=45% for different aspect ratio.

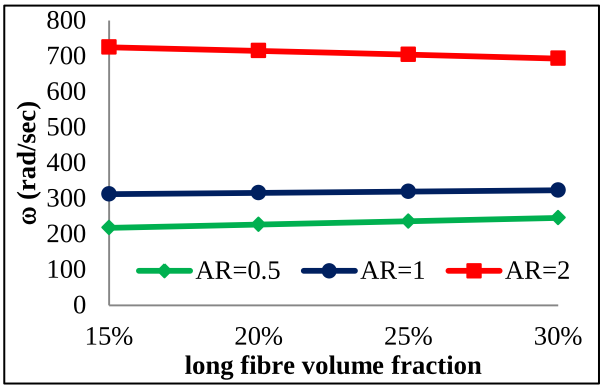

Fig. 22. Natural frequency vs. long fiber volume fraction for overall reinforcement voulme fraction(fiber and powder)$\forall_f$=50% for different aspect ratio.

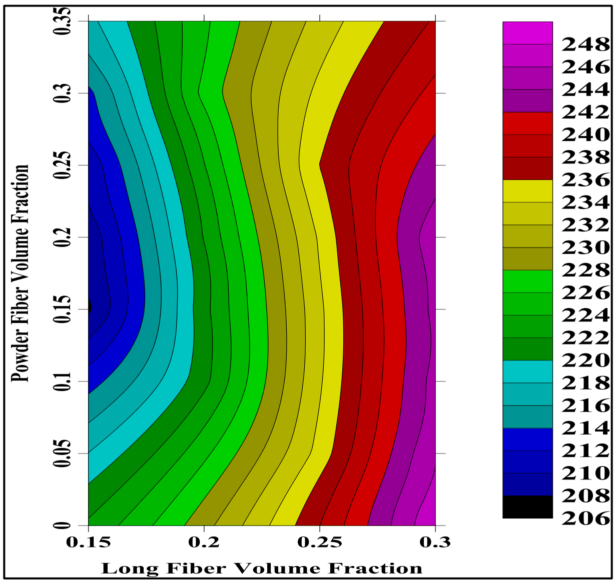

Fig. 23. Contour plot of the natural frequancy with respect to the fiber and powder volume fraction for AR=0.5.

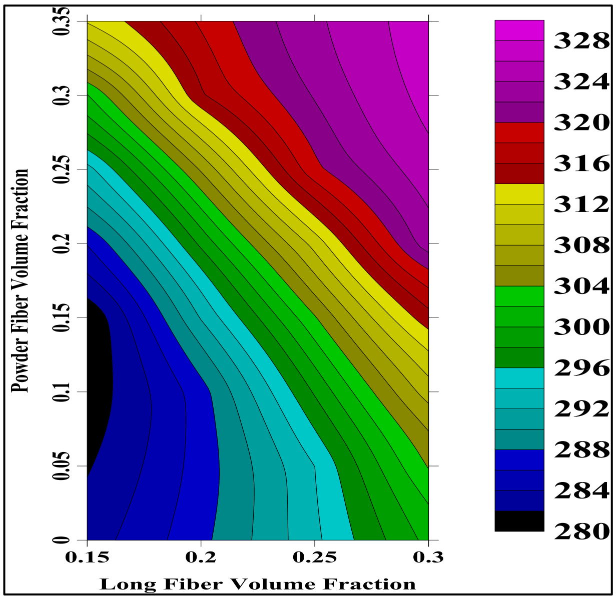

Fig. 24. Contour plot of the natural frequancy with respect to the fiber and powder volume fraction for AR=1.

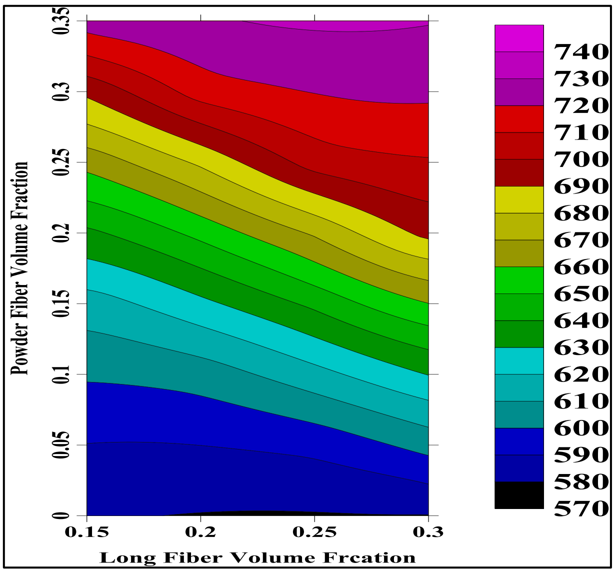

Fig. 25. Contour plot of the natural frequancy with respect to the fiber and powder volume fraction for AR=2

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