Longitudinal and Lateral Dynamics Control of a Small Scale Helicopter Using Adaptive Backstepping Controller under Horizontal wind Gusts

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Abstract- **This paper considered the problem of stabilization of longitudinal and lateral dynamics of an unmanned autonomous helicopter (UAH) for hovering flight mode in a gusty environment. The controllers design is based on generic linear model which successfully describe the behavior of most small-scale helicopters. A backstepping design procedure is used to design the adaptive controller for longitudinal and lateral dynamics based on control Lyapunov approach. For comparison purposes we also design another controller based on the linear quadratic regulator (LQR) criteria. The simulation results demonstrate that the proposed adaptive controllers can effectively attenuate the gust effects and achieve rapid and accurate longitudinal and lateral position in the gusty environment.**

1. INTRODUCTION

Among variety of Unmanned Air Vehicles (UAVs), unmanned autonomous helicopters constitute one of the most versatile and agile platforms for the development of autonomous flight systems. A helicopter can operate in different flight modes, such as vertical take-off/landing, hovering, longitudinal/lateral flight, and bank to turn which gives them the advantage of effective observation from various positions. Among these abilities hovering and vertical take-off ability are necessarily needed. Unmanned autonomous helicopter control system should make these performances achievable by improving the tracking performance and disturbance rejection capability in different weather conditions. Therefore, robustness is one of the critical issues which must be considered in the control system design for small unmanned autonomous helicopter, especially those covering large flight envelop. One major problem rarely addressed by researchers to date is that of a wind disturbance. To cope with such a problem researchers have considered different approaches such as robust feedback linearization, robust nonlinear H∞ controller [1], neural network [2], robust backstepping controller [3] etc.

In this paper, we proposed an adaptive backstepping controller to stabilize the longitudinal and lateral position of the small scale helicopter in the presence of wind gusts. The backstepping control is a recursive procedure which is used in order to provide a consistent methodology for the reference trajectory. To do so, we exploit the fact that the structure of the system is in strict feedback form. We can observe that the backstepping recursive methodology is not used to stabilize the system but to create a reference trajectory model. With this consistent way you can systematically construct the error dynamics and then apply a variety of control designs for linear and nonlinear systems. Actually using backstepping controller we stabilize the error dynamics of the system. A major advantage of backstepping control is the construction Lyapunov function whose derivative can be made negative definite by a variety of control laws rather than by a specific control law by other control method. A robust H∞ control method of the longitudinal and lateral dynamics of the BELL 205 helicopters in the presence of model uncertainty is presented in [4]. A robust feedback method for helicopter stabilization to reject wind disturbance is presented in [5], wherein the wind disturbance is assumed to be the sum of fixed number of sinusoids with unknown amplitudes, frequencies and phases. In [6], the authors present a robust backstepping technique of an autonomous scale helicopter subject to parameters uncertainties and uniform time varying tridimensional wind gusts. With the assistance of an unknown input observer technique (UIO), the controller is reported to be able to handle the effect of these uncertainties on the autonomous helicopter. In [7] the authors propose a nonlinear H∞ horizontal position controller for hover and automatic landings of a RUAV in the presence of horizontal wind gusts. Control of a model scale helicopter under vertical wind gusts is discussed in [8]. The typical level of wind gust in this paper is less than 1m/s. Furthermore, the authors present an active disturbance rejection control strategy based on a nonlinear observer. In [9], the authors propose an H∞ robust controller for stabilize the altitude and attitude tracking in the presence of vertical wind gusts.

Taken into account such external wind disturbances in the model equations, we use the control Lyapunov function approach to show that our proposed backstepping controller is robust with respect to external wind gusts. For comparison purposes we design another controller based on the linear quadratic regulator (LQR) criteria. The altitude and yaw dynamics are controlled through the classical PID controllers. The performances of the controllers are simulated and compared in keeping the horizontal position constant in the presence of external wind gusts. The results show that the proposed adaptive control using backstepping method outperforms than the LQR.

The rest of this paper is organized as follows. Section 2 briefly introduces the mathematical model used. Section 3 presents the gusts model. Section 4 presents the control structure of the adaptive backstepping controller as well as LQR controller. Section 5 discusses simulation results. Finally, the paper is concluded in Section 6.

2. Helicopter Linear Model

Among other UAVs, Unmanned Autonomous Helicopter (UAH) has specific characteristic; the helicopter can move vertically, float in the air, turn in place, move forward and lateral and can perform these movements in combinations. Because of this, helicopter dynamic modeling is a very complex problem. In 6DOF form, the motion states and control inputs are represented as

$$x = \{u, w, q, \theta, v, p, r, \phi\}$$
$$u_c = \{\delta_{col}, \delta_{lat}, \delta_{lat}, \delta_{ped}\}$$

The variables u, v and w represent the helicopter linear velocity in body frame; p, q and r denote roll, pitch and yaw rates respectively; and ϕ, θ represent roll, pitch attitude respectively. A conventional single main rotor helicopter has four independent control inputs, δ_{lat}, δ_{lon}, δ_{col} and δ_{ped} which denote the deflection of the lateral cyclic, longitudinal cyclic, main rotor collective pitch and tail rotor collective pitch respectively. The collective commands control the magnitude of the main rotor and tail rotor thrust and other two control commands control the inclination of the Tip-Path-Plane (TPP) on the longitudinal and lateral direction.

Before getting into the control law design longitudinal and lateral dynamics should be analyzed. For this purpose, the nonlinear form of a helicopter equations of motion are given as follows,

$$\dot{u} = (vr - wq) - g\sin\theta + X/m \tag{1}$$

$$\dot{v} = (wp - ur) + g\sin\phi\cos\theta + Y/m \tag{2}$$

$$\dot{w} = (uq - vp) + g\cos\phi\cos\theta + Z/m \tag{3}$$

$$\dot{p} = qr(I_{yy} - I_{zz})/I_{xx} + L/I_{xx} \tag{4}$$

$$\dot{q} = pr(I_{zz} - I_{xx})/I_{yy} + R/I_{yy} \tag{5}$$

$$\dot{r} = pq(I_{xx} - I_{yy})/I_{zz} + N/I_{zz} \tag{6}$$

where, forces $f_b = [X, Y, Z]^T$ and moments $M=[L, R, N]^T$ are expressed in the body frame, m is the helicopter mass and I_{xx}, I_{yy} and I_{zz} are the moment of inertial about x, y and z axis. In order to complete the system, we need to the following two equations which relate the Euler angle rates to the angular velocity [10]

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{7}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{8}$$

We know that the cyclic longitudinal and lateral tilt of the main rotor disk is controllable through the cyclic pitch. Therefore, in this model, the longitudinal and lateral flapping dynamics can be represented by the first order equations as follows [11]:

$$\dot{a}_1 = -q - \frac{a_1}{\tau} + \frac{1}{\tau}\left(\frac{\partial a}{\partial u}u + A_{lon}\delta_{lon}\right) \tag{9}$$

$$\dot{b}_1 = -p - \frac{b_1}{\tau} + \frac{1}{\tau}\left(\frac{\partial b}{\partial v}v + A_{lat}\delta_{lat}\right) \tag{10}$$

where, δ_{lat} and δ_{lon} are the lateral and longitudinal cyclic control inputs, a_1 and b_1 are the lateral and longitudinal flapping angles, A_{lon} and B_{lat} are effective steady-state longitudinal and lateral gains from the cyclic inputs to the main rotor flapping angles and $\tau = \frac{16}{\gamma\Omega}$ is the rotor time constant with γ denoting the lock number and Ω is the main rotor angular speed. Linearization is essential to derive simplified working models, considering inherent instability under hover and slow flight conditions. The Dihedral effect is

$$\frac{\partial a}{\partial u} = -\frac{\partial b}{\partial v} = \frac{2}{\Omega R}\left(\frac{8C_T}{\alpha\sigma} + \sqrt{\frac{C_T}{2}}\right)$$

where R is the main rotor radius, σ solidity ratio, α lift curve slope and C_T the thrust coefficient. Now after linearizing equations (1-8) we get following parameterized model of decoupled longitudinal and lateral dynamics.

$$\begin{bmatrix} \dot{u} \\ \dot{q} \\ \dot{\theta} \\ \dot{a}_1 \end{bmatrix} = \begin{bmatrix} X_u & X_q & -g & X_a \\ M_u & M_q & 0 & M_a \\ 0 & 1 & 0 & 0 \\ \frac{A_u}{\tau} & -1 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} u \\ q \\ \theta \\ a_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{A_{lon}}{\tau} \end{bmatrix} \delta_{lon} \quad (11)$$

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{\phi} \\ \dot{b}_1 \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & g & Y_b \\ L_v & L_p & 0 & L_b \\ 0 & 1 & 0 & 0 \\ \frac{B_v}{\tau} & -1 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} v \\ p \\ \phi \\ b_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{A_{lat}}{\tau} \end{bmatrix} \delta_{lat} \quad (12)$$

Remark 1: Control inputs in the controller design process are set to be longitudinal and lateral flapping angles. They will be converted later into longitudinal cyclic and lateral cyclic for implementation.

3. Gust Model

For an analysis of a RUAV, wind gusts can be treated as either random (spectral turbulence) or discrete. For random gusts, typical spectral models include the Von Karman and Dryden turbulence models. The Von Karman model has been widely considered the more "realistic" model when it comes to defining turbulence spectra. However, due to the computational complexity of the Von Karman model, the Dryden model is typically used in aerospace vehicle analyses. There are many sources for wind models based upon empirical data that consist of passing band limited white noise through appropriate forming filters. The turbulence models are scaled with respect to RUAV altitude, velocity and wing span. The filters used to generate the Dryden spectral model of atmospheric disturbance [12] are given by:

$$H_u(s) = \sigma_u \sqrt{\frac{2L_u}{\pi U}} \frac{1}{1 + \frac{L_u}{U} s} \quad (13)$$

$$H_v(s) = \sigma_v \sqrt{\frac{L_v}{\pi U}} \frac{1 + \frac{\sqrt{3}L_v}{U} s}{\left(1 + \frac{L_v}{U} s\right)^2} \quad (14)$$

where, U is the true speed of a RUAV, σ_u, and σ_v are the root mean square intensities of the turbulence and L_u, L_v and L_w are the turbulence scale lengths that describe the behaviour of the wind gusts. In this work, the scale of turbulence, L_u, and L_v are assigned constant values of $L_u = L_v = 722.5m$ and $L_w = h$. And for low altitude region (altitude < 1000ft) the σ_u, σ_v, and σ_w turbulence intensities are given by

$$\sigma_w = 0.1W_{20} \quad (15)$$

$$\frac{\sigma_u}{\sigma_w} = \frac{\sigma_v}{\sigma_w} = \frac{1}{(0.177 + 0.000823h)^{0.4}} \quad (16)$$

where, W_{20} is the wind speed at 20 ft (6m) above the ground and can be approximated by U and altitude is described by h. In this paper, we consider typical levels of wind speed is and altitude as 3 *m/s* and -2 *m*, respectively.

4. Controller Design

In this section, we design the controller for the longitudinal and lateral dynamics of the helicopter dynamics separately and each based on an appropriate decoupled model. And for that a robust backstepping control method-based on Lyapunov's function for the horizontal position stabilize of a small scale helicopter is presented through the control of longitudinal and lateral flapping angles as the control inputs. But for comparison purposes we also design another controller based on the linear quadratic regulator (LQR) criteria. The linear motion equations for longitudinal and lateral dynamics are described through Eq. (11-12). In the longitudinal dynamics (11), the parameter a_1 is a function of u and q. Similarly in the lateral dynamics (12), the parameter b_1 is a function of v and p. Hence, we cannot carry out the flight control laws via backstepping considering a_1 and b_1 as the control input. A common simplification practice, presented in [13] is to neglect the effect of the lateral and longitudinal forces produced by the flapping angles. These parasitic forces have a minimal effect on the translational dynamics compared to the propulsion forces produced by the stability derivatives X_θ and X_ϕ (in (11) and (12) are denoted by −g and g, respectively). We also neglect the stability derivative terms X_q and Y_p for deriving controller, because they are much smaller than the propulsion forces produced by the stability derivatives X_θ and X_ϕ .This assumption is physically meaningful and results into a linear system in feedback form.

According to our design purposes the helicopter should be separated into two interconnected subsystems. The first subsystem accounts for longitudinal mode and second subsystem accounts for lateral mode. As indicated in the above, the effect of the translational forces produced by the flapping motion of the main rotor is parasitic and negligible compared to the main source of propulsion, which are the forces produced by the roll and pitch attitude change of the fuselage. By neglecting the effect of the parameters *Xa* , *Yb*, X_q and Y_p the longitudinal-lateral dynamics will have strict feedback forms. As a result the simplified description of the longitudinal mode is given as,

$$\dot{x} = u\,, \quad \dot{u} = X_u\, u - g\,\theta$$

$$\dot{\theta} = q\,, \quad \dot{q} = M_u\, u + M_q\, q + M_a\, a_1$$

Similarly the simplified description of the lateral mode is given as,

$$\dot{y} = v\,, \quad \dot{v} = Y_v\, v + g\,\phi$$

$$\dot{\phi} = p\,, \quad \dot{p} = L_v\, v + L_p\, p + L_b\, b_1$$

A. Longitudinal Dynamics

In this subsection, the robust backstepping controller based on Lyapunov method is designed for the longitudinal dynamics in the presence of wind gusts. Linear model equations of the longitudinal dynamics under external disturbance are rewritten as:

$$\dot{x} = u\,, \quad \dot{u} = X_u\, u - g\,\theta\,, \quad \dot{\theta} = q$$

$$\dot{q} = M_u\, u + M_q\, q + M_a\, a_1 + \varepsilon$$

Step1: The design process starts with the definition of the longitudinal position error i.e.,

$$z_1 = x - x_d\,, \ \dot{z}_1 = \dot{x} - \dot{x}_d\,, \quad \dot{z}_1 = u \tag{17}$$

We consider u as a virtual control input and define u_d as a virtual control law for (17). Let z_2 be an error variable representing the difference between the actual value of u and its desired value u_d i.e.,

$$z_2 = u - u_d\,, \quad u = z_2 + u_d$$

Therefore, the resulting error signal is $\dot{z}_1 = z_2 + u_d$. At this stage we would like to design a virtual control law u_d which would make $z_1 \to 0$ as $t \to \infty$. Now, consider a control Lyapunov function

$$W_1 = \frac{1}{2} z_1^{\,2}$$

And its derivative as,

$$\dot{W}_1 = z_1 \dot{z}_1\,, \quad \dot{W}_1 = z_1 (z_2 + u_d)$$

We can now select an appropriate virtual control law u_d which would make $\dot{W}_1 \le 0$. A possible choice is $u_d = -k_1 z_1$ then, $\dot{W}_1 = -k_1 z_1^{\,2} + z_1 z_2$.

Clearly if $z_2 = 0$ then $\dot{W}_1 = -k_1 z_1^{\,2} \le 0$

where, k_1 is a scalar parameter which can be used to tune the output response. Now consider the time derivative of u_d as

$$\dot{u}_d = -k_1 \dot{z}_1\,, \ \dot{u}_d = -k_1 u \tag{18}$$

Step2: We derive the error dynamics for $z_2 = u - u_d$ and its time derivative as follows,

$$\dot{z}_2 = \dot{u} - \dot{u}_d\,, \quad \dot{z}_2 = x_u u - g\theta + k_1 u$$

In which θ is viewed as a virtual control input. Now define a virtual control law θ_d and let z_3 be an error variable representing the difference between actual and virtual control inputs i.e., $z_3 = \theta - \theta_d$. Therefore,

$$\dot{z}_2 = (x_u + k_1) u - g(z_3 + \theta_d)$$

Now choose a control Lyapunov function as follows,

$$W_2 = W_1 + \frac{1}{2} z_2^{\,2}\,.$$

Computing its time derivative one obtains,

$$\dot{W}_2 = \dot{W}_1 + z_2 \dot{z}_2$$
$$\dot{W}_2 = -k_1 z_1^{\,2} + z_2 \{z_1 + (x_u + k_1) u - g\theta_d\} - g z_2 z_3$$

We can now select an appropriate virtual control θ_d to cancel out some terms related to z_1, z_2 and u, while the term involving z_3 cannot be removed. $\theta_d = g^{-1}\{z_1 + (x_u + k_1) u + k_2 z_2\}$
(19)

Therefore, $\dot{W}_2 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} - g z_2 z_3$.

Clearly if $z_3 = 0$ then $\dot{W}_2 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} \le 0$.

Now the time derivative of θ_d as

$$\dot{\theta}_d = g^{-1}\{\dot{z}_1 + (x_u + k_1)\dot{u} + k_2 \dot{z}_2\}$$
$$\dot{\theta}_d = g^{-1}\{u + (x_u + k_1)(x_u u - g\theta) + k_2 \dot{z}_2\}$$

Step3: We derive the error dynamics for $z_3 = \theta - \theta_d$.

And its time derivative as, $\dot{z}_3 = \dot{\theta} - \dot{\theta}_d$

$$\dot{z}_3 = q - g^{-1}\{u + (x_u + k_1)(x_u u - g\theta)\} \\ - g^{-1}k_2\{(x_u + k_1)u - g(z_3 + \theta_d)\}$$

In which q is viewed as a virtual control input. Now define a virtual control law q_d and let z_4 be an error variable representing the difference between actual and virtual control inputs i.e., $z_4 = q - q_d$

Therefore, $$\dot{z}_3 = z_4 + q_d - g^{-1}\{u + (x_u + k_1)(x_u u - g\theta)\} \\ - g^{-1}k_2\{(x_u + k_1)u - g(z_3 + \theta_d)\}$$

Now choose a control Lyapunov function,

$W_3 = W_2 + \frac{1}{2} z_3^{\;2}$. And its time derivative as follows,

$$\dot{W}_3 = \dot{W}_2 + z_3 \dot{z}_3$$
$$\dot{W}_3 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} - g z_3 z_2 + z_3[z_4 + q_d \\ - g^{-1}\{u + (x_u + k_1)(x_u u - g\theta)\} - g^{-1}k_2\{(x_u + k_1)u \\ - g(z_3 + \theta_d)\}]$$

We can now select an appropriate virtual control q_d to cancel out some terms related to z_1, z_2, z_3 and u, while the term involving z_4 cannot be removed.

$$q_d = g z_2 + g^{-1}\{u + (x_u + k_1)(x_u u - g\theta)\} + \\ g^{-1}k_2\{(x_u + k_1)u - g(z_3 + \theta_d)\}] - k_3 z_3 \qquad (20)$$

And the derivative of q_d as follows,

$$\dot{q}_d = g\dot{z}_2 + g^{-1}\{\dot{u} + (x_u + k_1)(x_u \dot{u} - g\dot{\theta})\} + \\ g^{-1}k_2\{(x_u + k_1)\dot{u} - g(\dot{z}_3 + \dot{\theta}_d)\} - k_3 \dot{z}_3$$

Suppose, $\dot{q}_d = f(z_2, z_3, u, \theta, \theta_d)$ (21)

Therefore, $\dot{W}_3 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} - k_3 z_3^{\;2} + z_3 z_4$. Clearly if $z_4 = 0$ then $\dot{W}_3 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} - k_3 z_3^{\;2} \le 0$

Step4: We derive the error dynamics for $z_4 = q - q_d$.

And its derivative as follows,

$$\dot{z}_4 = \dot{q} - \dot{q}_d,\ \dot{z}_4 = M_u u + M_q q + M_a a_1 + \varepsilon \\ - f(z_2, z_3, u, \theta, \theta_d)$$

In the above equation the actual control input appears. Our objective is to design the actual control input 'a_1' such that z_1, z_2, z_3, and z_4 converge to zero as t→∞.

Now, choose a Lyapunov function W_4 as, $W_4 = W_3 + \frac{1}{2} z_4^{\;2}$.

And its time derivative as follows,

$$\dot{W}_4 = \dot{W}_3 + z_4 \dot{z}_4$$
$$\dot{W}_4 = -k_1 z_1^{\;2} - k_2 z_2^{\;2} - k_3 z_3^{\;2} + z_3 z_4 + \\ z_4(M_u u + M_q q + M_a a_1 + \varepsilon - f(z_2, z_3, u, \theta, \theta_d))$$

We are finally in the position to design control input 'a_1' by making $\dot{W}_4 \le 0$ as follows,

$$a_1 = -M^{-1}{}_a(z_3 + M_u u + M_q q + \hat{\varepsilon} - f(z_2, z_3, u, \theta, \theta_d) \\ + k_4 z_4)$$
(22)

where, ε is an unknown parameter which is estimated as $\hat{\varepsilon}$. We define the parameter estimation error signal is

$$\hat{\varepsilon} - \varepsilon = \tilde{\varepsilon}.$$

After that,

$$\dot{z}_4 = -z_3 - k_4 z_4 - \tilde{\varepsilon} \qquad (23)$$

Again consider a Lyapunov function which is used to augment the estimated parameter error,

$$W_5 = W_3 + \frac{1}{2} z_4^{\ 2} + \frac{1}{2\gamma} \tilde{\varepsilon}^2$$

And its time derivative as follows, $\dot{W}_5 = \dot{W}_3 + z_4 \dot{z}_4 + \tilde{\varepsilon} \frac{1}{\gamma} \dot{\hat{\varepsilon}}$

$$\dot{W}_5 = -k_1 z_1^{\ 2} - k_2 z_2^{\ 2} - k_3 z_3^{\ 2} - k_4 z_4^{\ 2} - \tilde{\varepsilon}\left(z_4 - \gamma\, \dot{\hat{\varepsilon}}\right)$$

where, γ is a positive constant that determines the convergence speed of the estimate. In order to render the non-positivity of the Lyapunov derivative of the above equation, we choose the adaptation law for the estimated parameter $\hat{\varepsilon}$ as follows,

$$\dot{\hat{\varepsilon}} = \gamma\, z_4 \qquad (24)$$

Then

$$\dot{W}_5 = -k_1 z_1^{\ 2} - k_2 z_2^{\ 2} - k_3 z_3^{\ 2} - k_4 z_4^{\ 2} \le 0 \qquad (25)$$

Now that the derivative of the final CLF is negative definite, the system will be stabilized at its equilibrium point.

B. Lateral Dynamics

In this subsection, the robust backstepping controller based on Lyapunov method is designed for the lateral dynamics in the presence of wind gusts. Linear model equations of the lateral dynamics under external disturbance are rewritten as:

$$\dot{y} = v$$

$$\dot{v} = Y_v\, v + g\,\phi$$

$$\dot{\phi} = p$$

$$\dot{p} = L_v\, v + L_p\, p + L_b b_1 + \delta$$

where δ is an unknown external disturbance which is estimated as $\hat{\delta}$. Using the similar procedure as like of the longitudinal dynamics we get the following control input for the lateral dynamics

$$b_1 = -L^{-1}{}_b(e_3 + L_v v + L_p p + \hat{\delta} - f(e_2, e_3, v, \phi, \phi_d) + c_4 e_4) \qquad (26)$$

And parameter adaptation rule is $\dot{\hat{\delta}} = \gamma\, c_4$ (27)

C. LQR Controller Design

In this subsection, we describe the design procedure of a LQR controller. LQR theory results in a linear control law which minimizes the integral over an infinite time interval of the weighted sum of the squares of the elements of the system state and control vectors. Consider the linear time-invariant system

$$\dot{x} = A x + B u$$

$$y = C x$$

wherein the state $x \in \Re^n$, the input control $u \in \Re^m$, and the measured output $y \in \Re^p$. The sate feedback control law has the form:

$$u = -K x \qquad (28)$$

where K is a $m \times n$ matrix which minimizes the following performance matrix:

$$J = \int_0^\infty (x^T Q\, x + u^T R u)\, dt \qquad (29)$$

In (29), Q and R are the positive definite (or positive semi-definite) weighting matrices which will balance the relative importance of the input and state in the cost function J that we are trying to optimize. The state feedback gain can be computed as follows,

$$K = R^{-1} B^T P$$

where P is a positive definite matrix obtained from the solution of the following algebraic Riccati equation:

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \qquad (30)$$

5. SIMULATION RESULTS

In this section, numerical simulation results are presented to investigate the performance of the propose controller for the longitudinal mode and lateral mode of a small scale helicopter based on simulation parameters consistent with those employed in real applications. Although derived based on the linearized model, the proposed controller must be shown effective and useful on the nonlinear model of the helicopter. To show the effectiveness of the proposed controller in a gusty environment we compare the performance of the proposed nonlinear adaptive backstepping controller with a linear quadratic regulator (LQR) controller. For the LQR control, the weighting matrices for both longitudinal mode and lateral mode are considered to be Q_{lon}=diag[0.1 0.1 0.5 0.6], R_{lon}=30 and Q_{lat}=[0.5 0.01 0.5 0.6], R_{lat}=80 respectively. The state feedback matrices for both longitudinal and lateral dynamics are K_{lon}=[-0.1421 0.1043 0.6046 -0.1414] and K_{lat}=[0.1169 0.0620 0.4469 0.0866]. The difference between two controls method are appeared in fig.1 where the tracking errors in X and Y positions are less by using backstepping controller compared with the LQR controller.

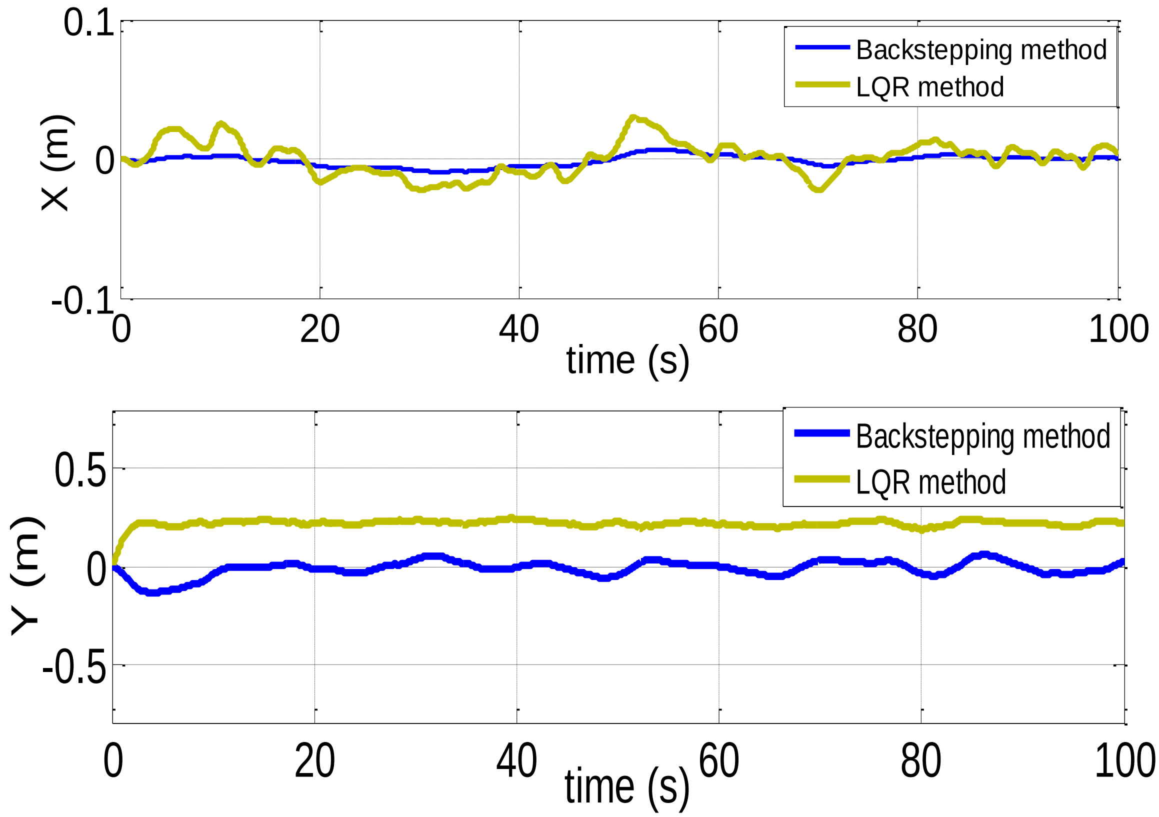


Fig. 1. Helicopter position response using backstepping and LQR

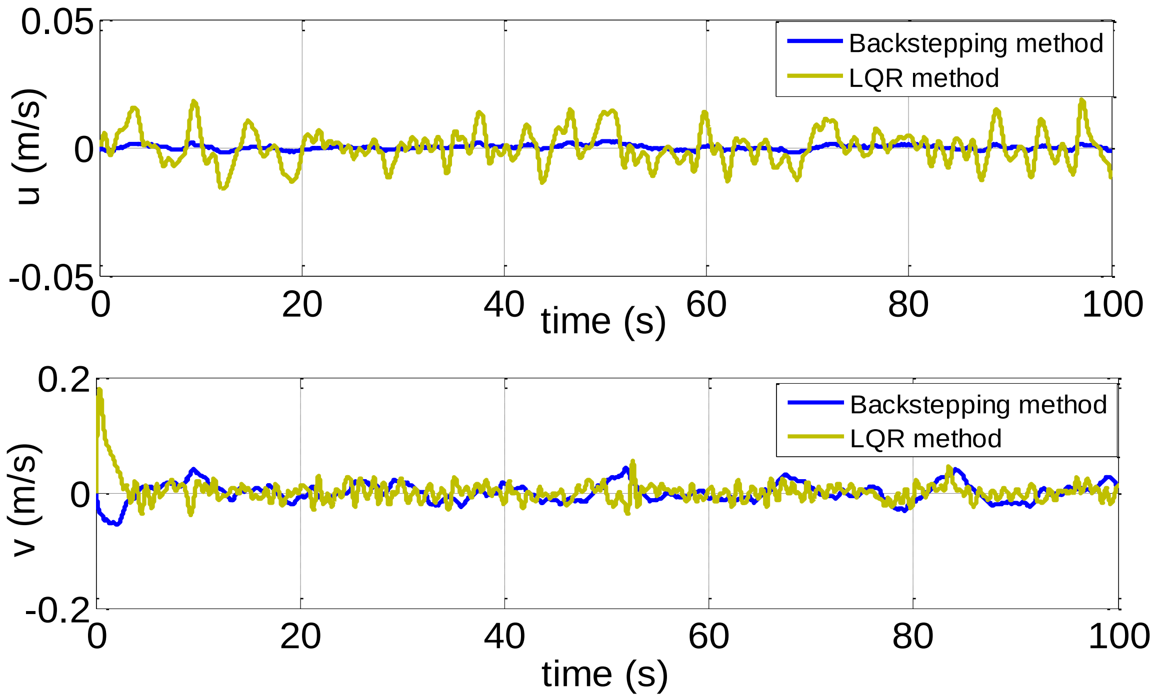
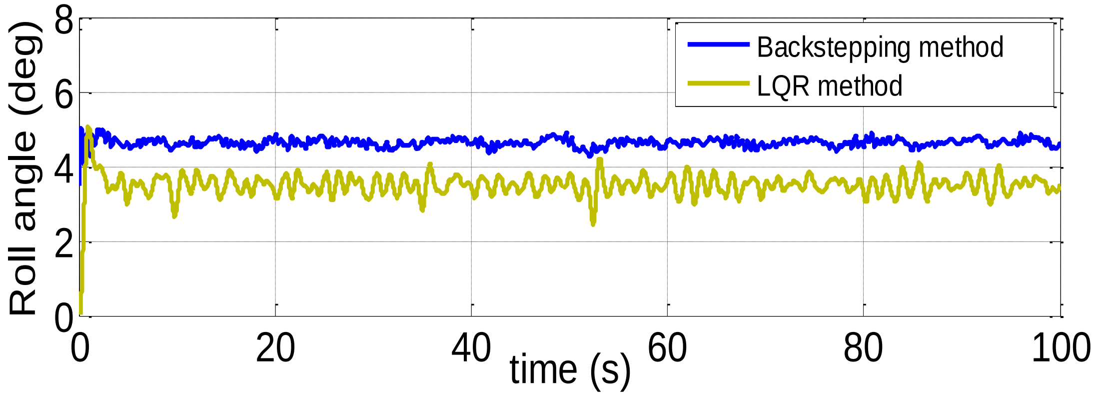


Fig. 2. Helicopter velocity response using backstepping and LQR



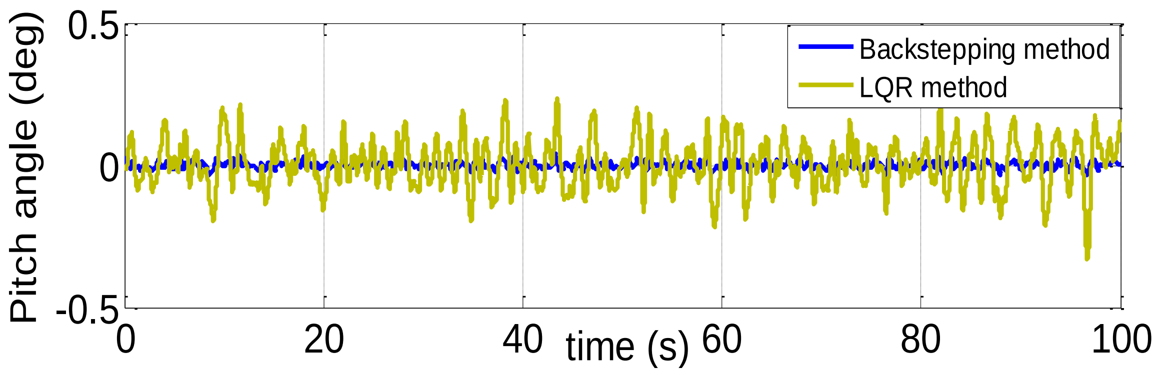


Fig. 3. Helicopter pitch response using backstepping and LQR

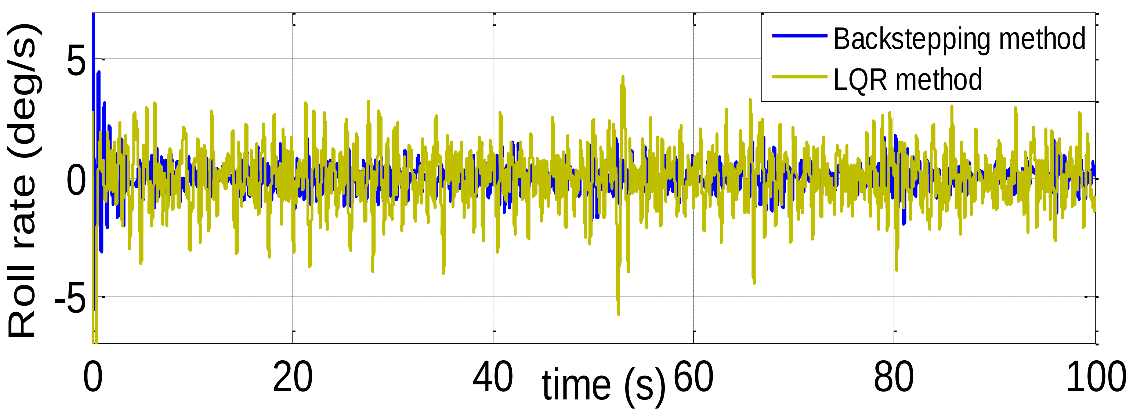


Fig. 4. Helicopter roll rate response using backstepping and LQR

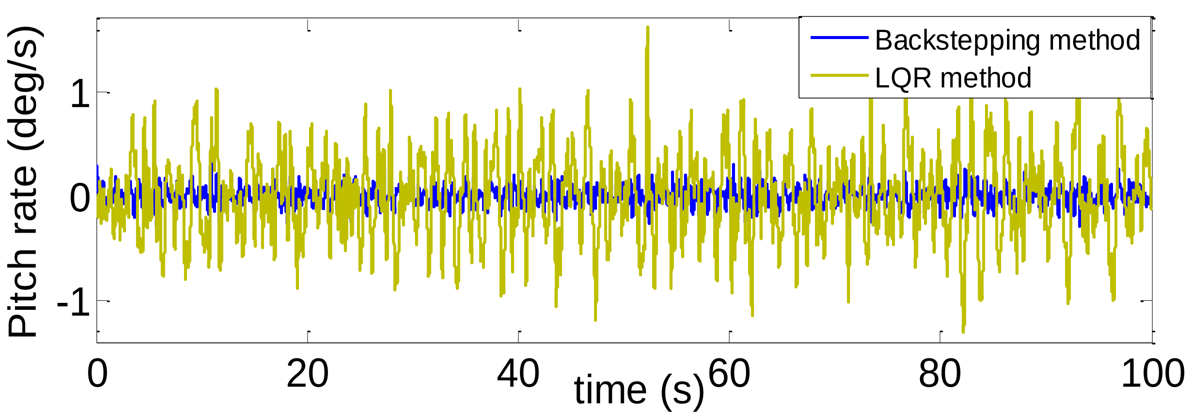


Fig. 5. Helicopter pitch rate response using backstepping and LQR

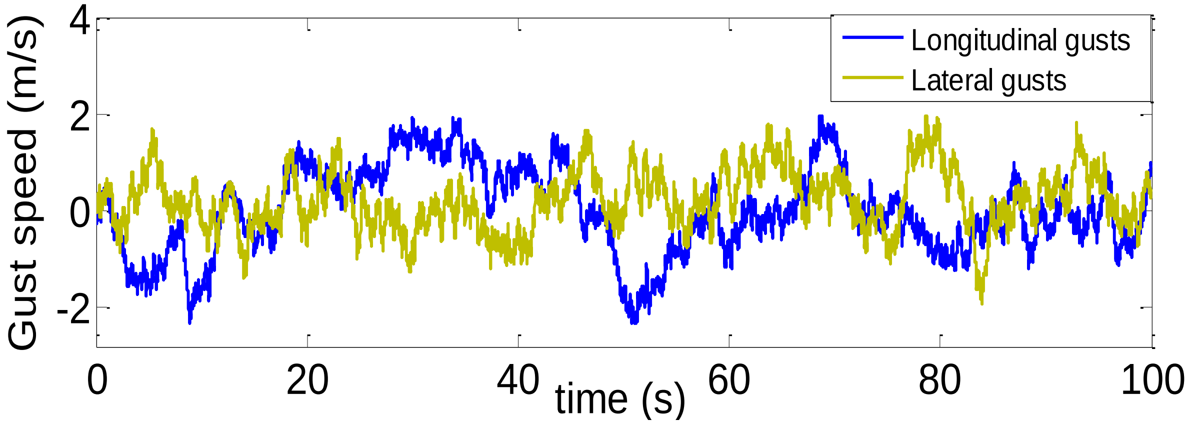


Fig. 6. Wind gusts to test controllers

The faster responses are the outcome of the rapid velocity responses depicted in Fig.2 and it is clear that velocities are converged quickly to zero, and are not subject to fluctuation as compared with LQR controllers. The sets of figures 4, 5 shows the results of the roll rate and pitch rate and we can see that roll rate and pitch rate are almost zero for backstepping controller while for LQR these are not zero. The results clearly demonstrate that the wind gust effect has been more effectively attenuated by backstepping controller than LQR. But considering the simulation performed, LQR controller also successful to stabilize the longitudinal and lateral position of the helicopter but as seen by the fact that helicopter cannot hover at the desired position (X=0, Y=0). So, our proposed adaptive backstepping controller is better than LQR controller in a gusty environment. With the help of adaptive backstepping control laws, external uncertainty is considered in system equation and compensated by control but in case of LQR controller external uncertainty is not considered in system model equation. The Dryden wind disturbance is shown in fig.6 that used in the simulation to test the controllers.

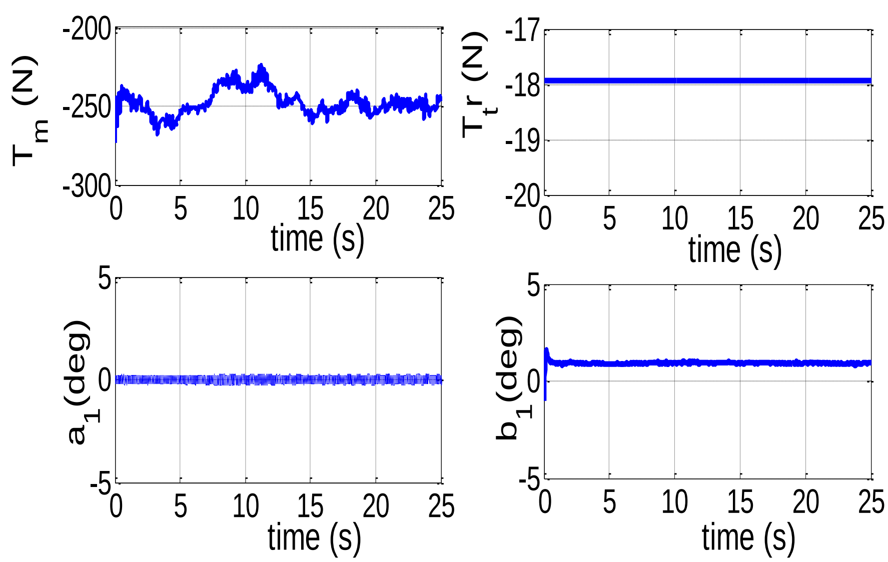


Fig. 3. Behaviour of the control inputs

The control inputs obtained using robust backstepping which do not exceed the constraints of the helicopter are shown in Fig.3.

6. Conclusion

In this paper, a Lyapunov's based backstepping control and LQR control have been applied for the control of longitudinal and lateral position of the small scale helicopter in the presence of wind gusts. Comparison simulation results show that the adaptive backstepping controller can settle the longitudinal and lateral position of the small helicopter more rapidly than a LQR controller in a gusty environment. But we can say that the LQR controls algorithm also produces satisfactory performances against external wind guts. Implementation of our proposed autonomous flight control method on the real system and flight test to prove their feasibility in real applications are remained for future work.

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