High precision CNC machining of axisymmetric die cavities

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***Abstract*— A high precision method is described for CNC milling of axisymmetric die cavities used to manufacture parts of revolution by processes such as casting, forging, injection, molding etc. The desired form of the cavities is achieved by selecting as generatrix curve (profile) any plane curve, implicitly or parametrically defined, which fulfills specific imposed by the user criteria. Tool center motion is generated by modeling each machining pass as a path composed of generatrix curve segments and semicircular arcs. The surface quality is controlled by keeping the distance between successive scallops within a programmed value. Each segment of the generatrix is processed by highly accurate real-time interpolation algorithm which generates cutting steps equal to the machine's resolution, while semicircular arcs are processed by the existing in all CNC systems standard circular motion. As it is shown, the whole machining task can be programmed in a single block of the part program. The effectiveness of the proposed system is verified by simulation tests for three representative applications.**

***Index Terms* — CAM, CNC machining, offset curves, motion generation, interpolation algorithms.**

I. INTRODUCTION

Despite the tremendous development in CNC technology, linear and circular motions continue to be the standard motions of CNC machines. Thus, if a particular shape cannot be programmed directly with these motions as are the cases of general curves and free form surfaces, a CAD/CAM system is engaged to approximate the specific shape with a series of small linear and circular segments, which are then encoded automatically into an executable voluminous CNC program [1]. However these approximations suffer from serious disadvantages. The sequence of the linear and circular segments approximating the desired path is traced by the system's linear and circular interpolator on a one at a time basis. Inevitably, their processing induces repeated accelerations and decelerations of the machine's motors, cause feederate fluctuations and consequently machining inaccuracies are raised with the machining time increasing substantially.

Unlike general free form surfaces, axisymmetric shapes are easily defined, since they are produced by the revolution of a plane curve, the generatrix, around a coplanar axis (Fig. 1). One needs only define the generatrix form and its position relative to the axis of symmetry. However, despite the particularity in the definition and the design of these surfaces, the available CAM systems deal with them as with free form surfaces. Consequently, the drawbacks mentioned in the previous paragraph continue to be present in the machining of these surfaces as well.

Following the present intention of research engineers to take advantage of the hardware capabilities of modern CNC systems by developing new real time CNC interpolators [2-5], the paper presents a manufacturing method for real-time machining on a 3-axis CNC milling machine of axisymmetric die cavities whose generatrix curve can be any plane curve, implicitly or parametrically defined. The method is characterized by it's accuracy, flexibility and simplicity.

Accuracy is obtained by applying the locus tracing concept [6] for driving the tool along the offset of the generatrix curve. The concept is generally applicable in motion generation and has been already applied successfully in a series of CNC machining tasks [7-9]. In this paper, its application is illustrated in the context of motion generation along the generatrix curve's offset. The concept avoids the complexity of using an exact analytic expression or a piecewise-analytic approximation for the offset [10-12] Instead, it uses analytic concepts and the defining geometric property to generate a succession of points on the locus (generatrix curve's offset), through repeated application of two analytically implemented construction operations. These operations are designed to achieve coordinate increment step control, automatic error control and maximum advance along the local tangent.

Flexibility is secured by the ability to modify easily, at the designing stage, the form of the generatrix curve until specific requirements (functional, aesthetic or other) of each particular situation are met.

Finally, simplicity is assured by the fact that the whole machining task can be programmed in a single block of the part program. In this block, the user specifies the desired form of the generatrix, the size and initial position of the cutter, the distance between scallops and the required cutting conditions.

II. OFFSET TRACING OF THE GENERATRIX

An accurate machining of the considered die cavities requires accurate offset cutter paths along the generatrix and

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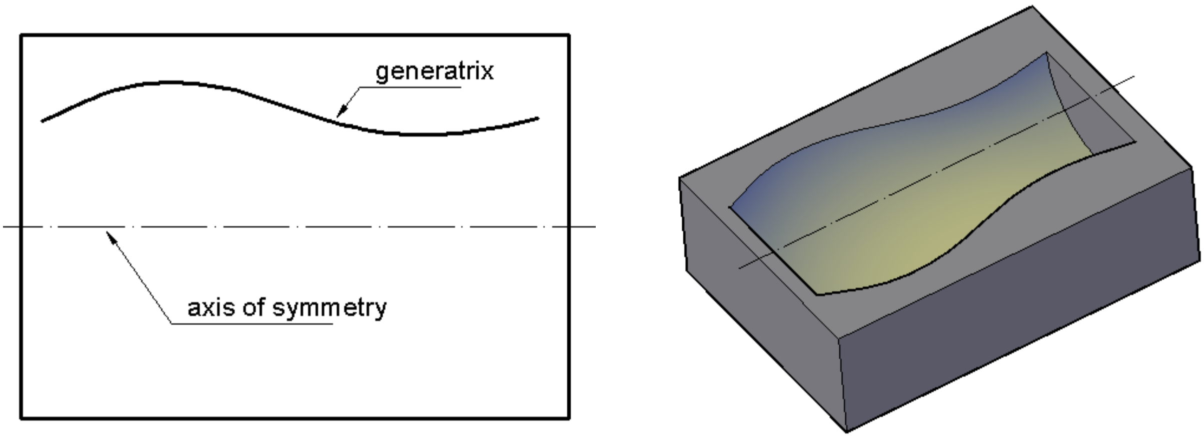

Fig. 1. Definition of an axisymmetric die cavity

it's mirror image. The generation of an accurate motion along generatrix curve's offset is treated as a locus tracing problem. The algorithm guides the tool-center through repeated application of two analytically implemented construction operations, maintaining exact contact (within 1 BLU[1]) along the entire path. In each iteration, the set of candidates steps is represented by the vector expression

$$dP = [dX, dY]$$
$$dX, dY \in [-1, 0, 1] \quad (1)$$
$$|dX| + |dY| \geq 1$$

The number of possible steps in each point is 8 (Fig. 2). The last inequality excludes the combination of zero values for both dX, dY, which does not constitute a step.

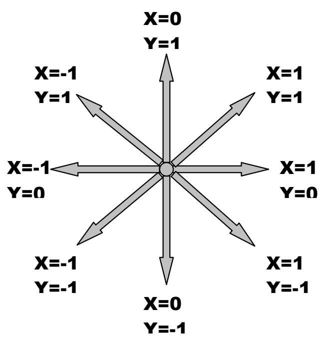

Fig. 2. The eight candidate steps

The best step is one, which maximizes the advance $\mathbf{T_i dP}$ (Fig. 3) along the local tangent $\mathbf{T_i}$ while, at the same time, it satisfies a criterion of proximity, which in our case is applied to the offset. Implementation of the proximity criterion requires the use of a proximity function, which, in the neighborhood of Pi, provides a measure of closeness to the offset. A suitable proximity function is derived from the fixed distance property of the offset

$$\begin{aligned} p &= (P - p_i)^2 - R^2 \\ &= (X - x_i)^2 + (Y - y_i)^2 - R^2 \end{aligned} \quad (2)$$

where R is the radius of the cutting tool.

Notice that for $\mathbf{P}$ lying on the offset p=0, while p increases absolutely as the distance of $\mathbf{P}$ from the offset increases.

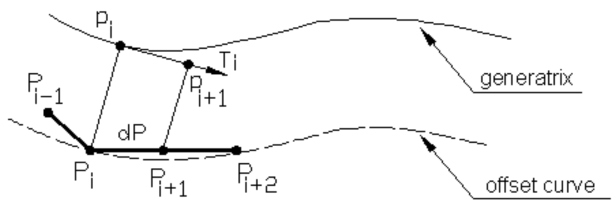

Fig. 3. Step selection process

Since the choice of step is limited to those prescribed by Eq.(1), the fixed distance property cannot be applied in a rigid manner. Rather, p is used as a proximity measure, from which a differential

$$\begin{aligned} \Delta p &= p_X dX + p_Y dY \\ &= 2(X_i - x_i)dX + 2(Y_i - y_i)dY \end{aligned} \quad (3)$$

can be developed, giving the effect of each candidate step on the position error. To satisfy the proximity requirement, dP must point towards the offset locus. In algebraic terms, it must drive the value of p towards 0. Specifically, it should give $\Delta p<0$ if $p>0$ and $\Delta p>0$ if $p<0$ or $p\Delta p<0$. Thus, if $\mathbf{T_i}$ is the local tangent vector, step selection is formulated as a constrained optimization problem

[1] BLU: Basic Length Unit. For a given CNC machine it represents the smallest distance the machine can resolve.

$$\begin{aligned}&\text{maximize } \mathbf{T_i dP}\\&\text{subject to } \Delta\mathbf{p} >=< 0\end{aligned} \tag{4}$$

where the sign > = < stands for $\geq$ when p<0 and for <0 when $p \geq 0$. A more explicit formulation can be obtained by introducing Eq.(3) and an expression of the local tangent vector. The latter depends on the functional representation of the generator entity. If the generator is given implicitly as f(x,y) = 0, then **Ti** = [f_Y, -f_X] and the step selection problem becomes

$$\begin{aligned}&\text{maximize } f_Y dX - f_X dY\\&\text{subject to } (X_i - x_i)dX + (Y_i - y_i)dY \; \substack{\geq\\<} \; 0\end{aligned} \tag{5}$$

For a parametric representation x=u(t), y=v(t), $\mathbf{T_i}$ =[u', v'] the problem to be solved is

$$\begin{aligned}&\text{maximize } u'dX + v'dY\\&\text{subject to } (X_i - u_i)dX + (Y_i - v_i)dY \; \substack{\geq\\<} \; 0\end{aligned} \tag{6}$$

Once an optimal step **dP** is determined from Eq. (5) or Eq. (6), the normality condition is enforced by throwing a normal from the newly selected point P_{i+1}= P_i + dP to the generator, to locate the next generator point p_{i+1}. This point is computed by solving the normality condition

$$T \cdot (P_{i+1} - p) = 0 \tag{7}$$

for p by Newton's method, using p_i to start the iterations. Again, the specific condition to be solved depends on the generator's representation. For an implicit representation, Newton's method is applied to the system

$$\begin{aligned}&f(x,y) = 0\\&f_Y(X_{i+1} - x) - f_X(Y_{i+1} - y) = 0\end{aligned} \tag{8}$$

to compute the coordinates $x=x_{i+1}$, $y=y_{i+1}$ of p_{i+1}, while for a parametric representation, the normality condition

$$u'(t)(X_{i+1} - u(t)) + v'(t)(Y_{i+1} - v(t)) = 0 \tag{9}$$

is solved for $t=t_{i+1}$ to determine the new point p_{i+1}=[u(t_{i+1}), v(t_{i+1})]. The new t is obtained as the root of (9), using Newton's iterative formula, which in this case takes the form

$$t_{i+1} = t_i - \frac{u'(t)\;(X_{i+1} - u(t)) + v'(t)\;(Y_{i+1} - v(t))}{u''(t)\;(X_{i+1} - u(t)) + v''(t)\;(Y_{i+1} - v(t)) - u'(t)^2 - v'(t)^2} \tag{10}$$

Two flow charts of the offset interpolator program, covering the respective modes of representation of the generator curve (implicit and parametric), are shown in Fig. 4.

III. TOOL PATH PLANNING AND PART PROGRAMMING

A vertical three-axis CNC milling machine is the appropriate system for machining the die cavities. In order to generate the desired shape, the cutter must be moved so as to remain tangent to the surface created by the the revolution of the generatrix. A tool commonly used for generating the specific form of surfaces is a spherical end milling cutter, which has the convenient property that the center of its spherical end remains at a constant distance from the generated surface, while the tool axis maintains a vertical orientation.

A. *Tool path planning*

A convenient tool center path consists of a series of small arcs of prescribed length along the generatrix curve’s offset, followed by semicircular motion perpendicular to the axis of symmetry, followed by arcs of equal length along the mirror image of the generatrix curve’s offset, until the end of the generatrix curve’s offset is reached (Fig. 5).

The interpolation program generates the necessary steps for the tool motion, using a separate routine for the steps along the generatrix curve’s offset or its image and for the steps along the semicircular segments of the path. The programmed distance between scallops (t) is used to determine when to switch from one routine to the other.

B. *Part programming*

The information needed for the machining are introduced in the part program via a statement of the form:

Gnn P01 X **P02** 60 **P03** 70 **P04** 5 **P05** 3 **P06** 100

where Gnn a non allocated G-code selects machining cycle for an axisymmetric cavity with a specific generatrix curve and P01,…P06 parameters specifying the following:

P01 – the axis of symmetry, X
P02, P03 – parameters specifying the form of the generatrix.
P04¬ – the offset distance, 5 (tool-radius)
P05 – the distance between scallops, 3
P06 – feedrate, 100 mm/min

The axis of symmetry in P01 must be one of the two axis of the machine table, X or Y.

Generally, in practical machining, the programming of surfaced parts consists of two machining sequences: rough and

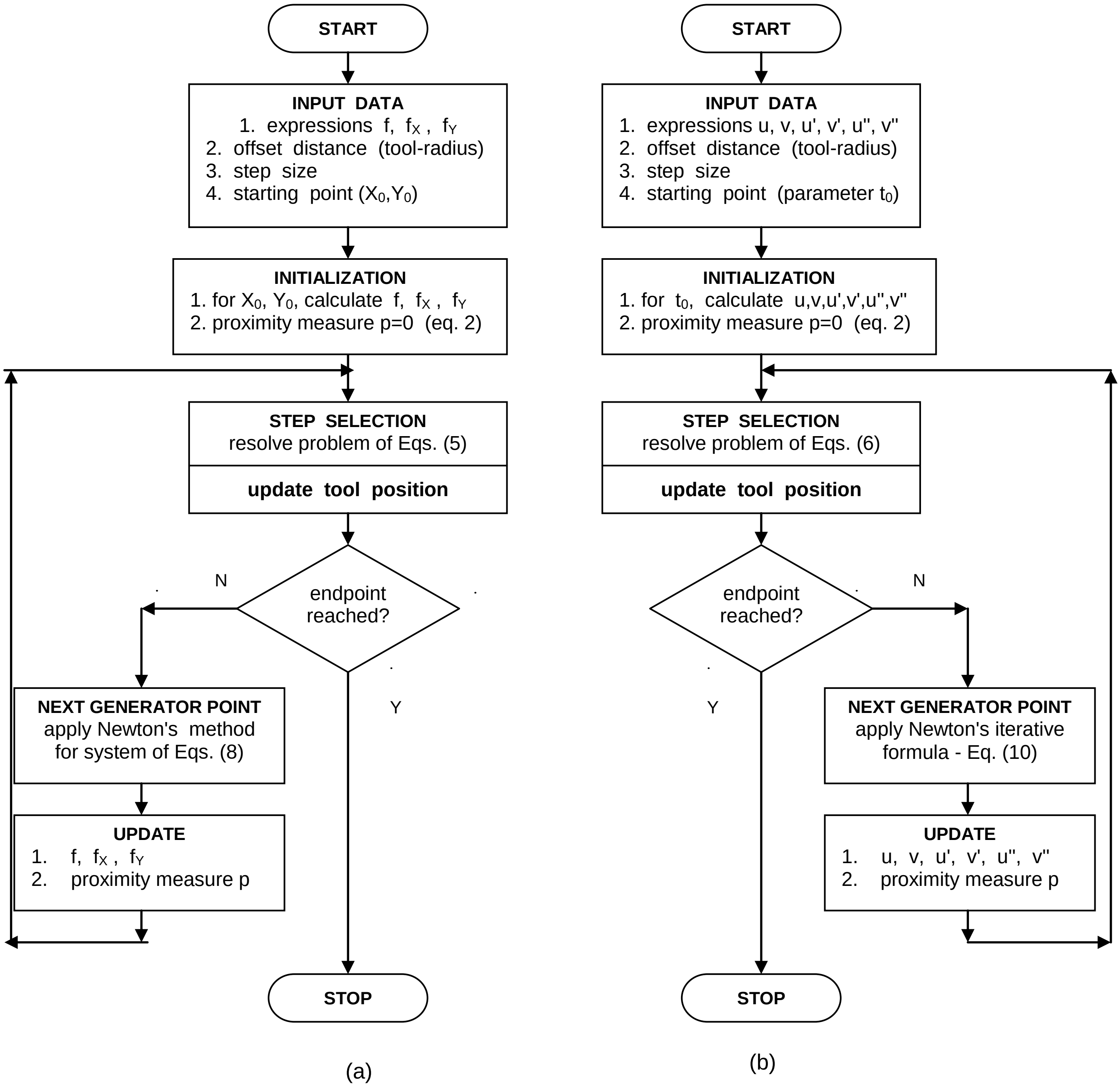

Fig. 4. Flow charts of the offset interpolator program. (a) for implicitly defined generators. (b) for parametrically defined generators

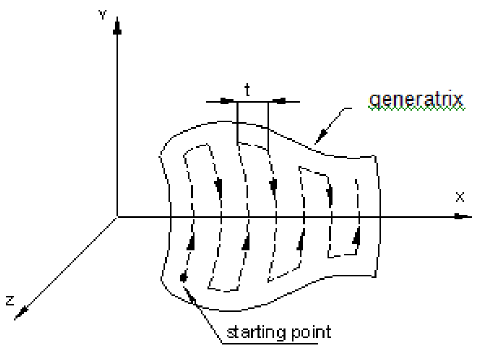

Fig.5. Tool center path for a revolved surface

finish. Rough machining removes most of the unwanted raw stock material while keeping the tool a safe distance from the part's surface. This rough pass is generated by the procedure outlined above, but using a tool radius figure R' □ R where R is the actual tool size. This leaves R' - R stock to be removed in the subsequent finish pass. The programmable distance between scallops is a factor offering the possibility to control the surface quality. The highest quality may be achieved by setting this distance equal to 1 BLU.

IV. Test results

To demonstrate the effectiveness and versatility of the proposed method, three representative plane curves were selected as generatrix curves of three axisymmetric dies respectively. These are a serpentine, a cycloid and an epitrochoid [13]. Based on their cartesian or parametric equations, the section, via certain numerical examples, describes the implementation mode of the method, suggests part programming codification and shows the simulated tool paths.

A. Application 1

1) *Cycloid as generatix curve:* Cycloid is one of the most famous curves in the history of mathematics. It was named by Galileo in 1599. The curve is described by a point P at distance b from the center of a circle of radius a as the circle rolls on the x axis. If the distance b is less than the radius a (b<a), the curve is as shown on Fig. 6 and is called *curtate* cycloid.

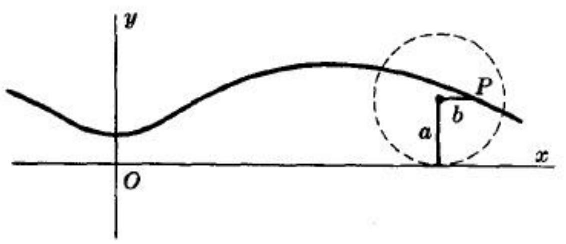

Fig. 6. Defnition of cycloid

The curve is parametrically expressed by equations

$$x = a.t - b.\sin t$$
$$y = a - b.\cos t \qquad (11)$$

where, t is the parameter corresponding to the angle through which the rolling circle has rotated, measured in radians.

The curve is also expressed by the Cartesian equation:

$$x = a.\cos^{-1}(1 - \frac{y}{a}) - \sqrt{y(2.a - y)}\,.$$
(12)□

2) *Numerical example:* Figure 7 shows the simulated tool path, as was recorded by the intrerpolator program, for an axisymmetric die cavity which has as generatrix curve a cycloid curtate with a=20 and b=8, parametrically defined as

$$u = 20\, t - 8 \sin t$$
$$v = 20 - 8 \cos t \qquad 13)$$
$$0 \le t \le 2.5\, \pi$$

with the following input data:
- starting and end point (expressed in terms of parameter t)
- tool radius R=10 mm,
- step size 0.1 mm
- distance between scallops t=6 mm.
- expressions u, v, u', v', u", v".

The same result can be obtained by the implicit version of the interpolator program using the implicit form of the curve and the respective required data.

Figure 8 shows the simulated tool path for the same die form with modified the distance between scallops (t=3 mm). As it can be seen, the smaller value of t leads to a bigger number of semi-circular cuts which in practice can ensure an improved surface quality.

3) *Part program codification:* As regards the codification of the specific machining in the framework of the part program and consistent to paragraph 3.1, it could be expressed by a statement of the form:

Gnn **P01** X **P02** 20 **P03** 15 **P04** 10 **P05** 6 **P06** 400

where,

Gnn – An attributed G code for machining axisymmetric cavities with cycloid as generatrix curve
P01 – the axis of symmetry, (X)
P02 – parameter specifying the **a** value, (20)
P03 – parameter specifying the **b** value, (8).
P04 – the tool-radius, (10mm)
P05 – the distance between scallops, (6 mm)
P06 – the feedrate, (400 mm/min)

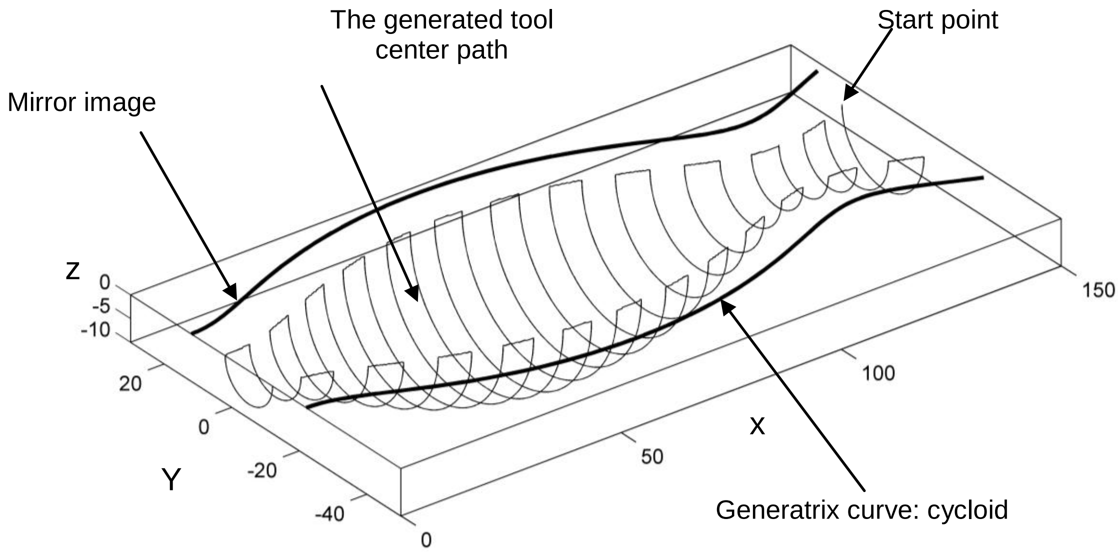

Fig. 7. The simulated tool center path for an axisymmetric die cavity with a cycloid as generatrix curve.

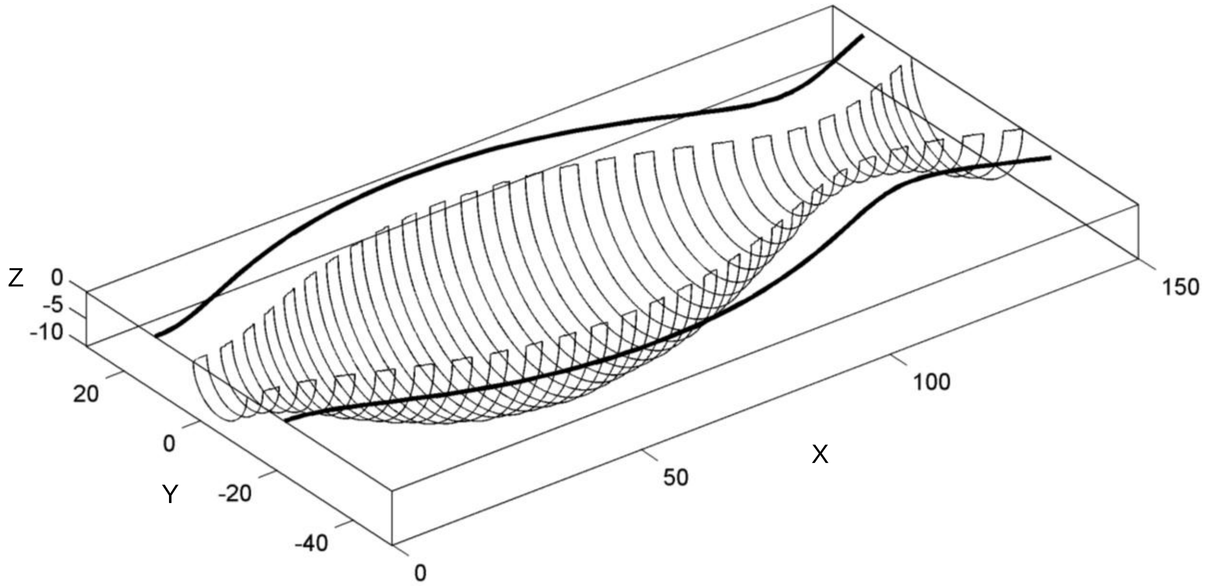

Fig. 8: The same die cavity as in Fig.7 with smaller distance between scallops

B. Application 2

1) *Serpentine as generatix curve:* Serpentine (Fig. 9), is a cubic curve with a beautiful geometry that has historically been used in many fields such as art, architecture, topography etc.

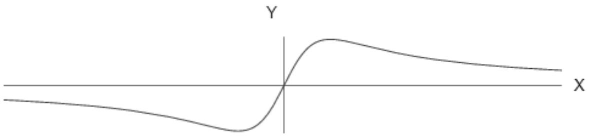

Fig. 9. Serpentine curve

The curve is described by Isaac Newton, and given by the cartesian equation

$$f(x,y) = x^2y + a^2y - abx = 0, \quad ab > 0 \tag{14}$$

Equivalently, it has a parametric representation

$$u = a\cot(t)$$
$$v = b\sin(t)\cos(t) \tag{15}$$

Using one of the two types of the curve's representations (eq. 14 or eq.15) and the respective version, implicit or parametric of the flowcharts shown in fig. 4, the interpolator program generates the required motion along it's offset.

As it can be seen from the flowcharts, beyond the selected tool radius, step size and starting point, the interpolator program needs as input, the values f, f_X , f_Y for the implicit case, and the values of u, v, u', v', u'', v'' for the parametric case. All these values are directly deduced either from eq. (11) or eq.(12).

2) *Numerical example:* Figure 10 shows the simulated tool path, as was recorded by the intrerpolator program (implicit version), for an axisymmetric die cavity which has as generatrix curve a serpentine curve with a=20 and b=15, implicitly defined as

$$f(x,y) = x^2y + 400y - 300x = 0 \tag{16}$$

with the following input data:
- starting and end point (expressed in terms of X and Y coordinates)
- tool radius R=4 mm,
- step size 0.1 mm
- distance between scallops t=4 mm.
- expressions f, f_X , f_Y.

The same result can be obtained by the parametric version of the interpolator program, using the parametric form of the curve and the respective required data.

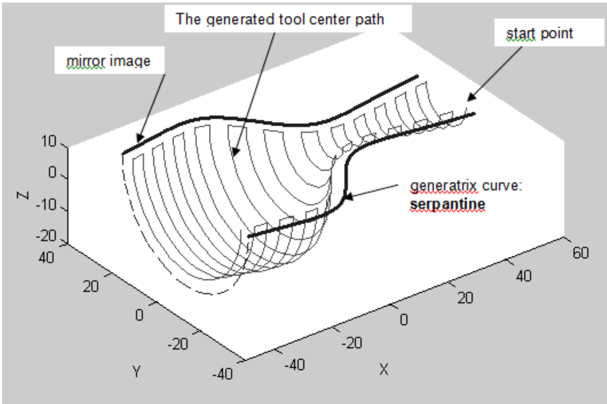

Fig. 10 The simulated tool center path for an axisymmetric die cavity with a serpentine as generatrix curve.

3) *Part program codification:* In this case, codification of the specific machining could be expressed by a statement of the form:

Gnn P01 X P02 20 P03 15 P04 4 P05 4 P06 400

where,
Gnn – An attributed G code for machining axisymmetric cavities with serpentine as generatrix curve
P01 – the axis of symmetry, (X)
P02 – parameter specifying the a value, (20)
P03 – parameter specifying the b value, (15).
P04¬ the tool-radius, (4mm)
P05 – the distance between scallops, (4mm)
P06 – the feedrate, (400 mm/min)

C. Application 3

1) *Epitrochoid as generatix curve:* An epitrochoid is a curve traced by a point P attached to a circle of radius r rolling around the outside of a fixed circle of radius R, where the point is at distance h from the center C of the exterior circle (Fig.11). The parametric equations for an epitrochoid are:

$$x = (R+r)\cos t - h\cos\left(\frac{R+r}{r}t\right)$$

$$y = (R+r)\sin t - h\sin\left(\frac{R+r}{r}t\right) \tag{17}$$

2) *Numerical example:* In this case a representative segment of an epitrochoid curve was selected as the generatrix of the axisymmetric die. The form of this segment is defined by the following values of the constants coefficients:

R= 40 mm
r = 80 mm (18)
h = 24 mm

with the parameter t varying between $\frac{2}{3}\pi \le t \le \frac{4}{3}\pi$

Thus, the parametric equations (17) of the epitrochoidal segment take the form:

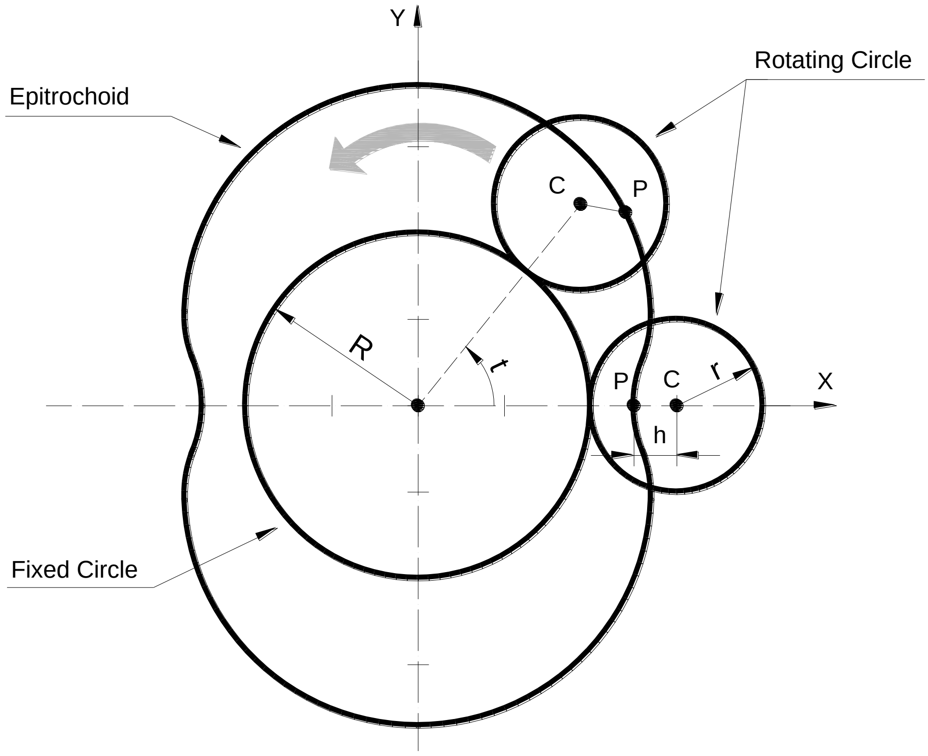

Fig. 11. Definition of epitrochoid.

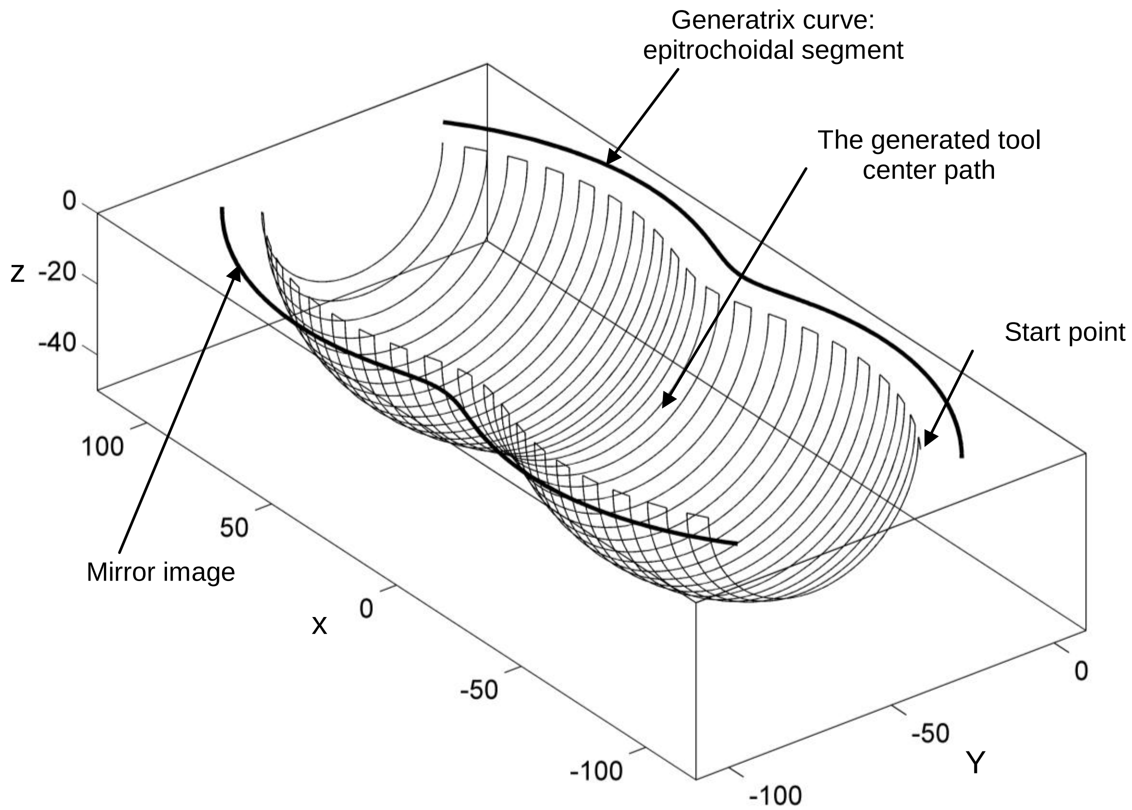

Fig. 11. The simulated tool center path for an axisymmetric die cavity with an epitrochoidal segment as generatrix curve.

$$u(t) = 150\cos t - 24\cos(3t)$$
$$v(t) = 150\sin t - 24\sin(3t) \quad (19)$$

Figure 12 shows the simulated tool path, as was recorded by the intrerpolator program, for an axisymmetric die cavity which has as generatrix curve the above epitrochoidal segment and the following input data:

- starting and end point (expressed in terms of parameter t)
- tool radius R=10 mm,
- step size 0.1 mm
- distance between scallops t=10 mm.
- expressions u, v, u', v', u'', v''.

3) *Part program codification:* In this case, codification of the specific machining could be expressed by a statement of the form:

Gnn P01 X P02 100 P03 50 P04 24 P05 4 P06 4 P07 400

where,

Gnn – An attributed G code for machining axisymmetric cavities with epitrochoid as generatrix curve
P01 – the axis of symmetry, (X)
P02 – parameter specifying the R value, (100)
P03 – parameter specifying the r value, (50).
P04 – parameter specifying the h value, (24).
P05¬ – the tool-radius, (4mm)
P06 – the distance between scallops, (4mm)
P07 – the feedrate, (400 mm/min)

V. Conclusions

A manufacturing method for the machining of axisymmetric die cavities whose profile can be any plane curve parametrically or implicitly defined was presented. The proposed method comprises tool path planning, real-time interpolation algorithm for motion generation along the generatix's offset and NC-codification for programming a 3-axis CNC milling machine that incorporates the appropriate software. Contrary to the approximation practice followed by CAM systems, the method provides a smooth motion without continuous start – stop cycles of the machine's motors. Simulation results have shown the effectiveness of the method in it's accuracy, flexibility and simplicity. The embodiment of the suggested algorithm in the control of a CNC milling machine could enhance it's feature generating capabilities since the potential applications of the presented algorithm are not limited to the three specific examples but can cover any other profile curve presenting interest in the design and manufacturing of axisymmetric dies.

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