Finite Element Investigation of Stress Concentration Factor in the Concrete Wall with Openings

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ABSTRACT: Widely known, that stress concentration in the building elements is a major factor influencing on the design and consequence on the cost. So objective of this paper is to investigate the stress concentration phenomenon in the concrete wall. For this reason we decided to study the stress concentration factor in the concrete wall, working in condition of plane stress under vertical load, distributed along above edge. In order to achieve this goal, elastic stress state of concrete walls with variety type of openings was modeled by finite element method and analyzed by using COMSOL program.

Key words: Stress Concentration Factor, Finite Element Method, Wall with Opening.

1. INTRODUCTION

IN this paper the stress concentration in the vertical loading wall without and with opening is discussed. This research focuses on stress concentration factor (SCF) of vertically loading concrete wall, since this is the stage that a typical wall would be subjected during its service life. In this paper we present the results of a study of the stress concentration in the concrete wall, working in condition of plane stress under vertical load, distributed along above edge.

Improvement of working properties, increasing of resources and reliability of structures are the most important challenge in front of the engineers. To succeed in solving this problem requires the systems approach to optimize all stages of analysis and design of all kinds of the structures. In this work, we touch on only a theoretical calculation of the walls with openings, and improve the accuracy of calculation.

The problem of stress concentration and the regions of stress concentration can be divided into two kinds:

1. The location of load application;
2. Changing of the geometry or physics of the solid body.
   i. Geometric discontinuities or stress raisers such as holes (openings), notches, and fillets;
   ii. Surface or internal irregularities (non-homogeneities) such as cracks.

Openings in general are areas of weakness and stress concentration, but needed essentially for lighting and ventilation. There are some norms and architectural recommendations about position of openings in wall. Stress concentration around door and window openings in the one hand, the maximum local stress around openings is usually more than the nominal stress, and on the other hand, it correlates to the maximum stress. Failure is predicted by the use SCF, $K_s$, which is determined in several manners. For simple geometries and loadings, it may be determined exactly using the theory of elasticity. For more complex problems, the stress concentration factor is determined numerically or experimentally. SCF, generally, is found in graphical form. The SCF around a opening (hole, notch, fillet…) in concrete walls has practical importance because it is commonly the cause cracks or failure. We know that the local failure around any door or window opening in wall can grow into the total failure of the wall. Therefore, the complete study of stress state of the wall, generally, and the concentration of the stress around openings, particularly, appears the main part of the analysis of such structures.

One of the objectives of application of SCF in this study is to achieve better balanced designs of concrete wall. This can lead to economy in expense of materials, cost reduction, and achieving lighter and more efficient configuration of walls and openings.

Nowadays, advanced computational programs and technology has made it possible to determine SCF for any geometric discontinuities or stress raisers such as holes (openings), notches, and fillets.

The purpose of this work is to demonstrate the existence of stress concentration in the vicinity of a geometric discontinuity in a vertically loaded concrete wall, and to obtain an approximate measure of the elastic (theoretical or geometric) stress concentration factor, $K_s$. In this case, the discontinuity is simply a square or rectangular through the concrete wall.

Under the stress concentration is meant abruptly local change in stress distribution in a deformed body, caused by different sources and factors: constructional (an abruptly change in the shape and dimensions of the cross sections, a discontinuity holes and cutouts, alien inclusions with mechanical properties differentiate from basal material, etc.); technological (high difference between mechanical properties of the material in the surface and in the volume of the structure, the presence of cracks); concentration of external factors (force, thermal); cracks occurred during the service.

A homogeneous plate has been investigated in last centuries. [1, 2, 3, 4]

The computational formulation for design of wall with potential stress concentration problems is intended to be used to assist in the design process. The high stress concentration at the edge of the opening is important in designing of wall. The SCF usually are determined either empirically or numerically using finite element methods.

Stress concentration is described by the following parameters:
theoretical SCF, which shows how many times the stress in the concentration zone exceeds nominal value, gradient of stress variation in the concentration zone, and etc.

Stress distribution around a hole depends on the stress condition. Kirsch (Kirsch, 1898) initially studied this problem for a single circular hole under a biaxial stress state.[1]

2. SCFS OF A 2D PROBLEM

One customary way to express the stress is by the net section SCF (Kt). Kt is the ratio of the maximum net section stress (σmax) to the nominal net section stress (σnom). The σmax is due to the stress concentration caused by the geometric features of the net section. For the axially loading members, the σnom is simply found by dividing the axial load (P) by the net section area (An) where An is the product of net section width (wn) and specimen thickness (t). Fig.1

![Fig.1 Plane Stress State](image)

SCF is function of:

i. the geometry or shape of the structure, but not its size or material;
ii. the type of loading applied to the member; (axial, bending, torsion, or combined)
iii. the shape, size and location of hole and opening; (fillet, radius, …)

Stress concentration in isotropic wall with different shape size, and position of cutouts are investigated, and this effect, the concrete wall is considered with square and rectangular, and the normal stress around openings in Y-direction is investigated.

An important feature of the phenomenon of stress concentration is that, while increasing the stress near the cutouts even uniaxial state may come into existence a complicated (2D or 3D) inhomogeneous stress state directly affects on the development of plastic deformation or fractural cracks. Thus, the bearing capacity of the main elements of many structures is usually determined by the stress state and conditions of strength in the concentration regions, because there is first and foremost comes the ultimate state and the destruction.

3. FINITE ELEMENT MODEL

The universal availability of general purpose structural analysis computer software has revolutionized the investigation of stress concentration. No longer are there numerous photoelastic stress concentration studies being performed.[8] The development of new experimental stress concentration curves has slowed to trickle. Often structural analysis is performed computationally in which the use of SCF is avoided, since a high-stress region is simply part of the computer analysis.

But the solid concrete wall under vertical distributed load (Fig.2) experiences normal stresses σx, σy, and shear stress τxy. The differential equations of equilibrium together with equation for these stresses in a plane elastic body are:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \bar{p}_x = 0
\]  

(1)

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \bar{p}_y = 0
\]  

(2)

\[
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\sigma_x + \sigma_y\right) = -f(v)\left(\frac{\partial \bar{p}_x}{\partial x} + \frac{\partial \bar{p}_y}{\partial y}\right) = 0
\]  

(3)

where \(\bar{p}_x, \bar{p}_y\) denote the components of the applied body force per unit volume in the x, y directions and \(f(v)\) is a function of Poisson's ratio:

\[f(v) = \begin{cases} 
1 + v & \text{for plane stress} \\
\frac{1}{1-v} & \text{for plane strain}
\end{cases}
\]  

(4)

In the present work it will be determined by the finite element methods (FEM) analysis.

The study of stress and strain state in the concentration zone is one of the most difficult problems of the theory of elasticity and plasticity.

This is due to:

- The complexity of circuit design elements in concentration zone (the complexity of the geometric model);
- The high intensity of the stresses and strains in the zone of the concentration and, consequently, the necessity to consider the physical and geometric nonlinearity of the elastic problem.
- Need to account of the plasticity and creep (the complexity of the model material). Still, many important aspects of the phenomenon of stress concentration are not comprehensive solution.

For the first time, the solution of the stress distribution around a circular hole for the planar elastic problem was obtained by Kirsch.[5]

The stress concentration around holes in thick plate in elastic range was studied by I. I. Vorovich and O. S. Malikina[6] and among the classical works we can denote the work of the Neyber G.[7]
discontinuities. This maximum local stress is many times greater than the nominal stress, $\sigma_{\text{nom}}$, of the member. Thus, the discontinuities cause areas of stress concentration within the component, and are often called “stress raisers”. In ideally elastic members, the ratio of the maximum stress to the nominal stress is designated, the theoretical stress concentration factor, $K_t$:[9]

$$\sigma_{\text{max}} = K_t \cdot \sigma_{\text{nom}}$$

$K_t$ - theoretical stress-concentration factor.

$\sigma_{\text{nom}}$ - nominal normal stress.

Similarly, we can also estimate the highly localized amplification of shear stress in the vicinity of a geometric stress concentration.

$$\tau_{\text{max}} = K_t \cdot \tau_{\text{nom}}$$

$K_t$ - theoretical stress-concentration factor for shear.

$\tau_{\text{nom}}$ - nominal shear stress.

The nominal stress of the above equations is typically derived from the elementary strength of materials equations, using either a net or a gross cross section.

The detailed study of the stress and strain state in stress concentration zones is an important prerequisite for optimal, efficient and reliable designs. Insufficiently rigorous evaluation of the stress state and strength conditions in the stress concentration regions during design may cause a fracture in the highly stressed zones of walls and lead to severe consequences. Highly relevant for the calculation of the walls of modern buildings is the study of stress concentrations in elastic-plastic deformation.

SCF can be derived through experimentation, analysis or computation:

1. Experimental Method. Optical method, such as photoelasticity is widely used for experimentally determining the SC at a point.[8] However, several alternative methods have been used: the brittle-coating method, grid method, strain gauge …


The optical methods are the most effective for experimental study of stress concentration. And the numerical methods, advisability, which have been seen for a long time, acquire a special significance at this time. On the one hand, this is associated with increased reliability requirements of modern engineering structures, the calculation of which should be based on the latest ideas about the behavior of the material in different loading conditions. It shall be ensured high accuracy of calculation. On the other hand, the emergence of electronic computing machines has led to a reassessment of the effectiveness of approximate methods associated with significant computing operations, which in the past limited their use.

Essentially, that the engineering calculations of strength, as a rule, do not require absolute accuracy. Moreover, it usually, make no sense, as much as, baseline data used in the calculation as material properties, acting loads, temperature modes, and etc. are not exact quantities. It’s clear that the calculations must be maintained approximation taking into account the degree of accuracy of initial and requirements applicable to the finite precision of calculations of the strength of the structures. All this speaks in favor of the numerical calculation methods.

With this model we can obtain qualitative and quantitative assessment of structural reliability under operational conditions simultaneously are solved two main problems:

1. Given an analysis of reliability of the structural under different load conditions and, also during service.

2. Optimization of structures for a given rate of reliability on purpose to obtain the minimum weight, maximum cost effectiveness, etc.

The first mathematical analyses of stress concentrations were published in 1937. Suggested form solutions are available for the simple stress states and not complex geometries. However, for more complex cases, experimental methods for measuring highly localized stresses (photoelastic tests, precision strain gage tests, membrane analogy for torsion, etc.) and computerized finite element solutions have been used. The results of the studies are available in the form of published graphs.[2,3]

It will be observed, that the stress concentration graphs are theoretical factors based on a theoretical homogenous, isotropic, and elastic material.

On this basis, the finite element models performed was aimed to investigate the SCF for wall with different shapes of openings, and to study and define the stress concentration factor for concrete wall with different openings at the middle part of the walls.

This work describes an analytical model, which was developed using the finite element program COMSOL, to investigate the analysis of SCF around openings in the concrete wall. The concrete wall is modeled in the COMSOL Multiphysics modeling environment with customized user interfaces and functionality optimized for structural analysis.

Previous research on walls’ SCF, in general, is extremely limited and focuses on empirical formulas.[2] To assess the strength under all conditions is important to determine, if it is possible, a more accurate view of the distribution of stresses and strains in the concentration region for specific states of the material: elastic, plastic and creep strains.

In this study we will examine the theoretical method (FEM) for determination of the true state of stress in the vicinity of stress raisers.

All problems investigated in the present work were modeled using Lagrange solid elements. Material properties: stiffness ($E$) of 25GPa and Poisson’s ratio ($\nu$) of 0.33. All presented models are loaded only in the Y-axis direction, with constant the thickness dimension oriented with the model’s z-axis (Thickness=10cm). The model’s x-axis therefore describes the width dimension, perpendicular to the loading direction. The problems investigated here vary in area and shape of hole. (Fig.2)
4. SOME REMARKS ABOUT THE STRESS CONCENTRATION AROUND OPENINGS

If the dimensions of a stress raiser are much smaller than those of the structural member, its influence is usually limited to a localized area or volume for a 3-dimensional case. That is, the global stress distribution of the member except for the localized area is the same as that for the member without the stress raiser. This kind of problem is referred to as localized stress concentration. Usually stress concentration theory deals with the localized stress concentration problems. The simplest way to solve these problems is to separate this localized part from the member, then to determine $K_t$ by using the formulas and curves (or as presented in this work by using FEM) of a simple case with a similar raiser shape and loading. If a wide stress field is affected, the problem is called nonlocal stress concentration and can be quite complicated. Then a full-fledged stress analysis of the problem may be essential, probably with general purpose structural analysis computer software.\textsuperscript{[3]}

In order to limit the size of the openings and identified the shape and position of these openings in concrete wall under vertical load need to be determined the vertical stress around opening. The model is proposed (Table 1 and 2) allows determining the location of the critical stress around opening in the concrete wall.

For the wall with openings, the maximum normal stress, $\sigma_n$, around holes depends on the size of the opening. For this purpose, the opening size factor is defined by Michael G. Allen and Yahya C. Kurama.\textsuperscript{[10]}

The explicated above is issue of the day for the calculation of walls with openings. The tendency to reduce the weight of the buildings leads to the necessity to use high-strength concrete, which, as a rule, has a higher sensitivity to stress concentration.

The most comprehensive source of stress concentration factors for commonly encountered geometries has been compiled by Peterson (1953, 1974).\textsuperscript{[2]} However in these references, the stress concentration factors for only filleted shafts and plates with holes are available and are only approximations based on photo elastic results for two-dimensional strips. The relation between two and three dimensional stress concentration factors is made by assuming an analogy exists between a circumferential fillet and a circumferential groove. This is the limitation of the Peterson Graphs for estimation of the SCF.\textsuperscript{[3]}

The numerical techniques are most effective due to advancement of high and large memory computers. These techniques can be applied for any minor change in the problem, which reduces the cost and time required for manufacturing and testing of several prototypes.

5. RESULTS AND DISCUSSION

However, when the wall contains discontinuity, such as shown openings (Table 1 and 2) sudden change in cross section, high localized stresses may also occur near the discontinuity. As shown in table 1, for wall with openings, high stress distribution will be at the chord along GH line of the openings where the cross sectional of the member will be at the least.

The figures and graphs in table 1 show the 3D deformed shape, distribution of normal stress $\sigma_n$ along Lines EF and GH, SCF $K_t$, depending to position of the square hole in concrete wall, and also comparison with the solid wall. The described FEM model was verified by comparing the results of stress along Y-axis for 5 types of wall, first
from which is the solid concrete wall (without opening) and others are with opening, as shown in table1. Thus, the comparison between SCF for wall with opening was done. The modeling, position, and dimensions of the openings in the finite element models are shown in table1. The FE analyses of the mentioned above walls show that the most critical section for $\sigma_y$ is line GH, in other words, the maximum stress locates near the opening along vertical line.

As we can see from table1 the location of the maximum normal stress for wall without opening is along the base of the wall, but for wall with opening is around opening along vertical line GH. The presented in table1 results show how the opening size affects on the distribution of the normal stress.

As noted above, the shape of the opening significantly, exerts on the kind and value of the normal stress in Y-axis direction (see results in table2). Based on the finite element analysis, the results of the stress state investigation depending to the shape of the cutouts are presented in table2. As we can see, when the size of the opening increases, the normal stress $\sigma_y$ around openings increases, consequently the SCF, $K_t$, increases.

This research provides an overview for the design of the required reinforcement in the location of maximum stress at all sides of the opening. Design and placement of reinforcement for wall with openings, particularly, described in Design of Rectangular Openings in Precast walls Under Combined Vertical and Lateral Loads.[10]

The FEA indicate that the directions of the principal stresses around opening are mostly horizontal. For symmetrical wall the stress at both right and left regions of opening are similar, but the concentration of stresses above opening differs from stresses below opening with connect to different loading and supporting condition.

The concentration of normal stress acting along section GH is not uniform, easily; we can observe that the stress at corners of the opening reaches its maximum magnitude. Basically, the research about SCF are related to the steel structures [11, 12, 13], but we know that for civil engineer, in addition to steel, the stress concentration in reinforced concrete and masonry structures is very important, for this reason, we tried to focus on the SCF in concrete wall.

6. CONCLUSION

1. The finite element model is used to conduct a stress investigation of concentration factor in concrete wall.
2. The parameters of studied are the opening size, shape, and position.
3. The variability investigation is studied to identify the stress concentration regions in the wall where maximum normal stresses occur.
4. The results indicate that the maximum stresses are around opening along vertical side of opening.
5. The presence of openings can affect the location and magnitude of the maximum normal stress around openings.
6. Using a computer program COMSOL allows more accurate approach to the problem of stress concentration.
7. This study shows that the size of the openings has a significant effect on $K_t$.
8. As can be seen from the presented results, the position of openings also affects on the values of normal stress and $K_t$.
9. Given that the main problem against the engineer - is to know the value of stress at any point in structure, and since this work gives a complete picture of the state of stress in each zone of the wall, leads us to a more efficient use of the material, but also helps control the shape of the openings and their position. As a consequence, leads to the economic feasibility of the structural analysis and design.
Table 1- Effect of the position of the opening on the distribution of the normal stress and SCF

<table>
<thead>
<tr>
<th>2D, $\sigma_y$</th>
<th>3D, $\sigma_y$</th>
<th>4</th>
<th>7.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>wall without opening</td>
<td>wall with opening</td>
<td>$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{normal}}}$</td>
<td></td>
</tr>
<tr>
<td>2.5m</td>
<td></td>
<td>Opening 1.5x2m</td>
<td></td>
</tr>
</tbody>
</table>
Table 1-Cont'd

<table>
<thead>
<tr>
<th></th>
<th>wall without opening</th>
<th>wall with opening</th>
<th>$K_v = \frac{\sigma_{\text{max}}}{\sigma_{\text{nominal}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D, $\sigma_y$</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>3D, $\sigma_y$</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>$\sigma_y$, along $EF$</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>$\sigma_y$, along $GH$</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td></td>
</tr>
</tbody>
</table>

H = 3m

$\sigma = \text{stress}$

$\text{L} = \text{length}$

$\text{al} = \text{along}$

$\text{no} = \text{no}$

$\text{min} = \text{minimum}$

$\text{max} = \text{maximum}$

$\text{K} = \text{Ratio}$
Table 1-Cont’d

<table>
<thead>
<tr>
<th>2D, σy</th>
<th>wall without opening</th>
<th>wall with opening</th>
<th>$K_i = \frac{\sigma_{\text{min}}}{\sigma_{\text{normal}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Wall without opening" /></td>
<td><img src="image" alt="Wall with opening" /></td>
<td>4.5</td>
</tr>
<tr>
<td>3D, σy</td>
<td><img src="image" alt="3D Stress" /></td>
<td><img src="image" alt="3D Stress" /></td>
<td>7.7</td>
</tr>
<tr>
<td>σy, along EF</td>
<td><img src="image" alt="Stress graph" /></td>
<td><img src="image" alt="Stress graph" /></td>
<td></td>
</tr>
<tr>
<td>σy, along GH</td>
<td><img src="image" alt="Stress graph" /></td>
<td><img src="image" alt="Stress graph" /></td>
<td></td>
</tr>
</tbody>
</table>
Table 1-Cont’d

<table>
<thead>
<tr>
<th>2D, $\sigma_y$</th>
<th>3D, $\sigma_y$</th>
<th>wall without opening</th>
<th>wall with opening</th>
<th>$K_r = \frac{\sigma_{\text{max}}}{\sigma_{\text{max of }}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$, along $EF$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_y$, along $GH$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$H = 3m$  
$H = 5m$  
$L = 5m$
Table-2 SCF and distribution of $\sigma_y$ around opening depending on the hole size.

<table>
<thead>
<tr>
<th>No</th>
<th>$\sigma_y$</th>
<th>Stress along horizontal line EF</th>
<th>Stress along horizontal line GH</th>
<th>$K_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Concrete wall 5x3m Without opening</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Opening1x1m</td>
<td><img src="image1" alt="Graph1" /></td>
<td><img src="image2" alt="Graph2" /></td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>Opening1x2m</td>
<td><img src="image3" alt="Graph3" /></td>
<td><img src="image4" alt="Graph4" /></td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>Opening2x2m</td>
<td><img src="image5" alt="Graph5" /></td>
<td><img src="image6" alt="Graph6" /></td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>Opening3x2m</td>
<td><img src="image7" alt="Graph7" /></td>
<td><img src="image8" alt="Graph8" /></td>
<td>2.4</td>
</tr>
</tbody>
</table>

Concrete wall 5x3m

Opening1x1m

Opening1x2m

Opening2x2m

Opening3x2m
REFERENCES


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   Email: soranrahim65@yahoo.com